Circuit model of a uniformly magnetized cylindrical permanent magnet

David Meeker
dmeeker@ieee.org

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Introduction

The problem of determining the operating point of a cylindrical permanent magnet is a recurring one. Since the flux density is not uniform inside the magnet, every part of the magnet operates at a slightly different operating point. However, it is useful to have a simplified model of a cylindrical magnet that assumes uniform operating point throughout the magnet. If properly formulated, this sort of model can provide insight about flux density in the magnet, energy stored by the magnet, and so on. This note will describe a simple circuit model of a magnet. To aid in the formulation of such models, a simple formula will be presented for the reluctance of the air traversed by the magnet circuit.

Magnet Model Assumptions

For the purposes of this note, it will be assumed that the magnet is an "ideal" magnet with an internal relative permeability of 1. In this case, the following simple formula gives the remanent flux density of the magnet as a function of energy product:

$$B_r = \frac{\sqrt{BH_{max}}}{5}$$

where BH_{max} is energy product in units of MGOe and the resulting B_r is in units of Tesla.

Circuit Model of a Magnet

A commonly used model has the form shown below in Figure 1. The model of the magnet itself consists of a flux source, ϕ_r, in parallel with the magnet's internal reluctance, R_m. Reluctance R_l represents the magnetic reluctance of the air section between the magnet's poles.

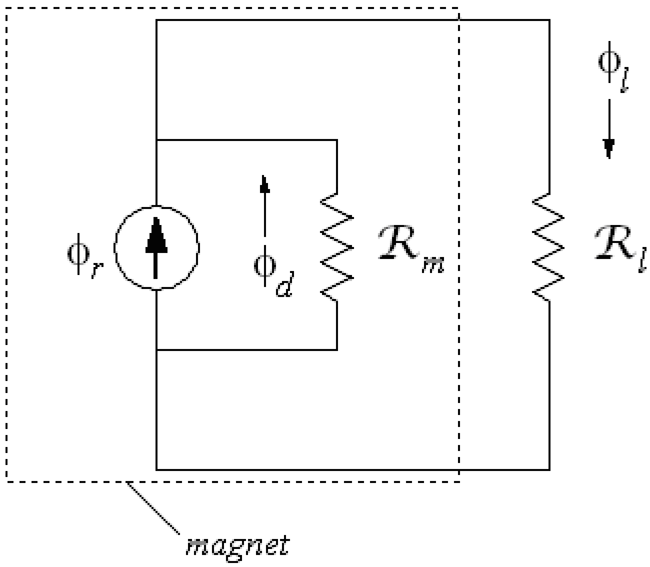


Figure 1: Magnet modeled as a flux source and parallel reluctance.

The internal reluctance of the magnet, R_m, is defined in terms of the geometry of the magnet:

$$R_m = \frac{l}{\mu_o a}$$

where l is the length of the magnet, a is the cross-section area of the magnet, and μ_o is the magnetic permeability of free space.

The remanent flux of the magnet is:

$$\phi_r = B_r a$$

where B_r is the remanent flux density of the magnet.

It is convenient to normalize the load reluctance by the internal reluctance of the magnet:

$$R_l = cR_m$$

where c is a dimensionless scaling factor that depends on magnet geometry. However, the value of c as a function of magnet dimensions is not intuitively obvious. An approximate but simple expression for c is derived later in this note.

Analysis of Magnetic Circuit Equations

Two equations can be written that describe the magnetic circuit. The conservation of flux in the circuit implies the first equation:

$$\phi_r + \phi_d = \phi_l$$

The second equation is the magnetomotive force (MMF) loop equation associated with the flux path that passes through the demagnetizing reluctance and back through the load reluctance. Since there is no MMF source in this loop, the total MMF drop must equal zero:

$$R_m \phi_d + c\, R_m \phi_l = 0$$

Solving these equations for demagnetizing and load flux yields:

$$\phi_d = -\left(\frac{c}{1+c}\right)\phi_r$$

$$\phi_l = \left(\frac{1}{1+c}\right)\phi_r$$

The factor relating the remanent flux to the demagnetizing flux is known in the literature as the "demagnetization factor", usually denoted as *N*:

$$N = \frac{c}{1+c}$$

Determination of the Demagnetization Factor

Historically, several different rationale have been employed to determine a suitable value of *N*. The selection of *N* on the basis of predicting the average demagnetizing field at the mid-magnet cross-section is called the "fluxmetric (or ballistic) demagnetization factor". The selection of *N* for the accurate prediction of the average demagnetizing field over the entire volume of the magnet is known as the "magnetometric demagnetization factor".

A simple way to compute a demagnetization factor similar to the magnetometric one is to choose the demagnetization factor on the basis of the numerically computed ratio of energy to coenergy. The FEMM finite element program will be used to compute the stored energy for various magnet aspect ratios, and an approximate but simple expression for the demagnetization factor will be deduced.

Magnetic Field Energy

To determine the relationship of energy and coenergy to the demagnetization factor, the energy and coenergy stored in the magnetic field of the permanent magnet must first be derived. The energy can be calculated by summing the energy stored in each reluctance:

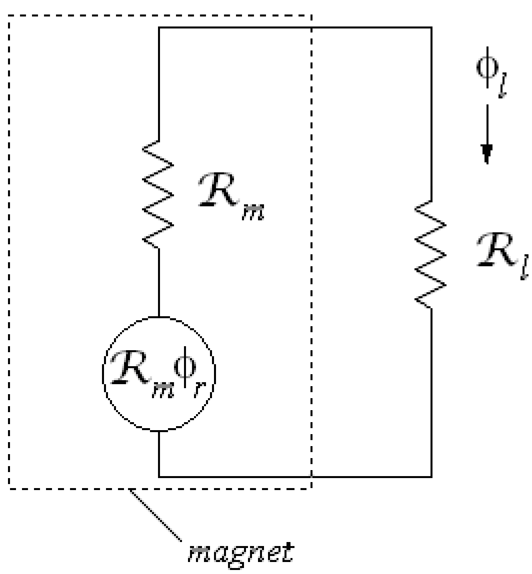
$$
\begin{aligned}
W &= \frac{1}{2}\left(R_m\phi_d^2 + cR_m\phi_l^2\right) \\
&= \frac{1}{2}\left(\frac{c}{1+c}\right)R_m\phi_r^2 \\
&= \frac{1}{2}NR_m\phi_r^2
\end{aligned}
$$

It is interesting to note the simple and direct dependence of the stored energy on the demagnetization factor.

Stored energy *W* can be directly interpreted as the ability of the magnet to perform mechanical work. When the magnet is shorted by a keeper, the demagnetization factor, and thus the stored energy, are both zero. Since there is no other source or sink for the stored energy, stored energy *W* must be delivered to the system as mechanical work when a keeper is applied.

Magnetic Field Coenergy

Coenergy is a useful quantity for the computation of forces in magnetic systems that combine permanent magnets and coils. The coenergy can be computed by considering the Thévenin-equivalent circuit of the permanent model. The Thévenin model is pictured in Figure 2. The coenergy is simply the energy deduced from the Thévenin magnet model.



The load flux is the same for both the Norton and Thévenin circuits so that it is straightforward to write an expression for the Thévenin circuit's energy, denoted *W*':

$$\begin{aligned} W' &= \frac{1}{2}(1+c)\, R_m \phi_l^2 \\ &= \frac{1}{2}\left(\frac{1}{1+c}\right) R_m \phi_r^2 \end{aligned}$$

Comparing the expressions for energy, *W*, and coenergy, *W'*, a straightforward method of computing the load reluctance factor, *c*, from finite element results is to evaluate the expression:

$$c = \frac{W}{W'}$$

where *W* and *W'* are obtained by integrating over the entire solution domain.

An OctaveFEMM function was created to automatically compute *c* for a given *l/d* (length/diameter) ratio. This function is discussed in detail in the Appendix. The results of this function, evaluated on a range from *l/d* =0.1 to *l/d* = 10. The results from the OctaveFEMM analyses are pictured below in Figure 3.

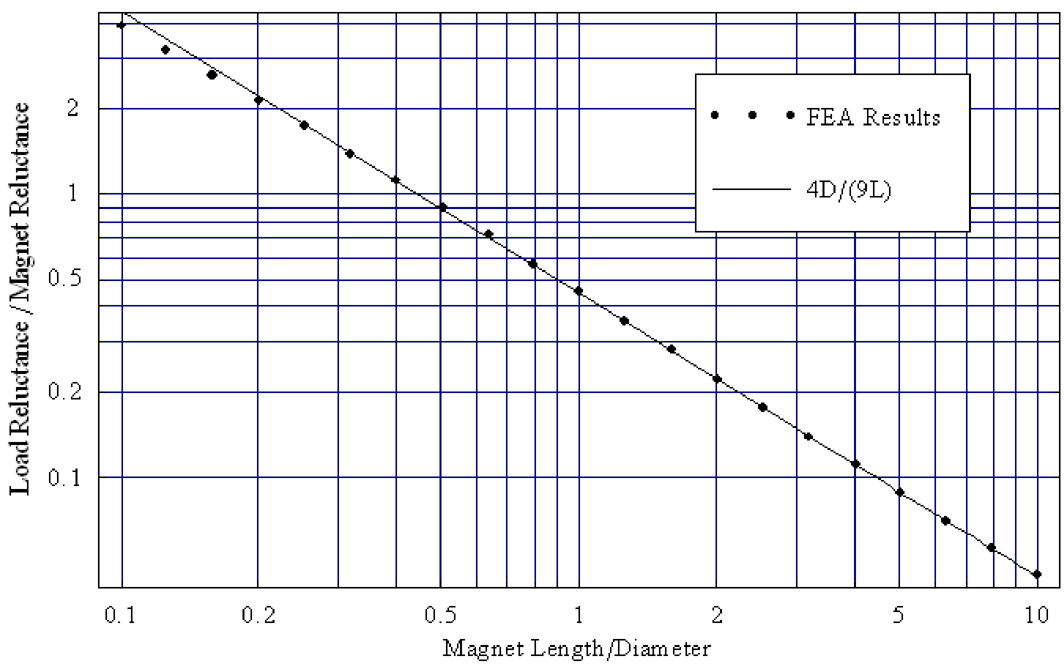


Figure 3: Load reluctance coefficient versus magnet *l/d* ratio.

After investigating various fits for the finite element results, it was found that the following expression is both simple and closely matches the load reluctance coefficient, *c*, over a wide range of magnet aspect ratios:

$$c = \frac{4}{9}\left(\frac{1}{l/d}\right)$$

The implied formula for demagnetization factor, *N*, is:

$$N = \frac{4}{4 + 9\ (l/d)}$$

This form shows good agreement with the tabulated values for magnetometric demagnetizing factor in [Chen1991], wherein a closed-form expression for the inductance of a single-layer coil containing elliptic integrals is used to generate the magnetometric demagnetization factor.

Conclusions

A simple circuit model of a cylindrical permanent magnet has been presented. Simple, approximate expressions for the reluctance of the air section of the magnetic circuit and the associated magnetometric demagnetization factor have been presented.

References

Chen, D.-X.; Brug, J.A.; Goldfarb, R.B., “Demagnetizing factors for cylinders,” IEEE Transactions on Magnetics 27(4):3601-3619, July 1991.

Appendix A: `getc.m`

This function computes the load reluctance factor, *c*, for a uniformly magnetized cylinder with a given length/diameter ratio.

```
function c=getc(ld)
      z=ld;
      ro=1/2;
      r=2*max([ro,z]);
      rx=1.1*r;
      openfemm;
      newdocument(0);
      mi_probdef(0,'inches','axi',1e-8,0,30);
      mi_drawrectangle(0,-z/2,ro,z/2);
      mi_drawarc(0,-r,0,r,180,5);
      mi_drawarc(0,-rx,0,rx,180,5);
      mi_drawline(0,-rx,0,rx);
      mi_addcircprop('icoil',1,1);
      mi_addblocklabel(ro/2,0);
      mi_addblocklabel((r+ro)/2,0);
      mi_addblocklabel((r+rx)/2,0);
      mi_addmaterial('magnet',1,1,10^6,0,0,0,0,1,0,0,0);
      mi_addmaterial('exterior',10,10);
      mi_addmaterial('air' ,1,1);
      mi_addboundprop('zero',0,0,0,0,0,0,0,0,0);
      mi_selectlabel(ro/2,0);
      mi_setblockprop('magnet',0,r/100,'<None>',90,0,1);
      mi_clearselected;
      mi_selectlabel((r+ro)/2,0);
      mi_setblockprop('air',0,r/100,'<None>',0,0,0);
      mi_clearselected;
      mi_selectlabel((r+rx)/2,0);
      mi_setblockprop('exterior',0,r/100,'<None>',0,0,0);
      mi_clearselected;
      mi_selectarcsegment(rx,0);
      mi_setarcsegmentprop(5,'zero',0,0);
      mi_saveas([tempdir,'tmp.fem']);
      mi_analyze;
      mi_loadsolution;
      mo_groupselectblock(0);
      energy=mo_blockintegral(2);
      coenergy=mo_blockintegral(17);
      c=energy/coenergy;
      closefemm;
```

The function creates a permanent magnet of the prescribed aspect ratio. The magnet is placed within a spherical shell with a somewhat magnetically permeable outer shell whose permeability is selected to give the same impedance as unbounded space (*i.e.* the edge region acts like a “first-order asymptotic boundary condition”). In FEMM, this type of impedance boundary condition is typically applied as a “built-in” boundary condition type at the boundary. However, some energy is stored in the external region implied by the asymptotic boundary condition. Implementing the asymptotic boundary condition “explicitly” allows access trivially to all problem regions for energy/coenergy integration so that accurate energy results are obtained. A picture of a typical solution is shown below in Figure 4.

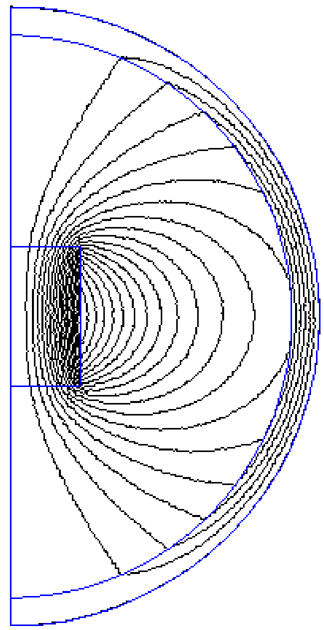


Figure 4: Solution region generated by the `getc.m` script for an *l/d* ratio of 1.