Improved Continuum Skin and Proximity Effect Model for Hexagonally Packed Wires

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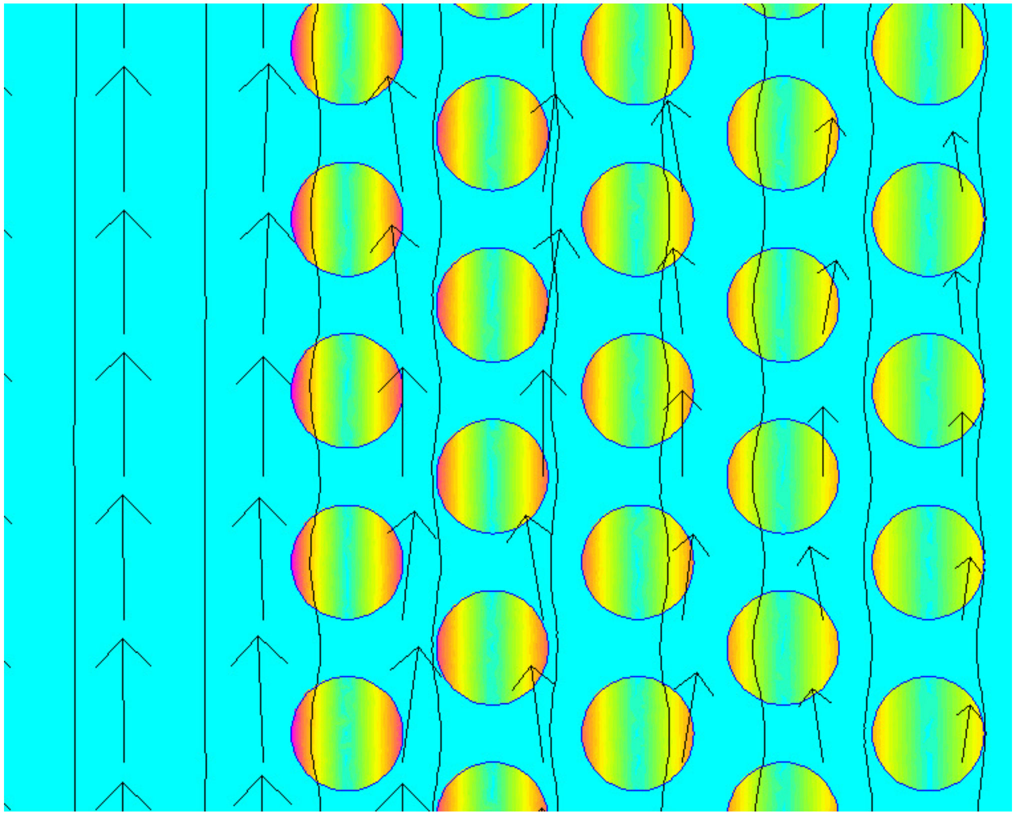
Induced currents

- When conductors are exposed to a time-varying magnetic field, currents are induced
- Losses that occur when conducting wires are exposed to a magnetic field
- Known as “Skin Effect” and “Proximity Effect”

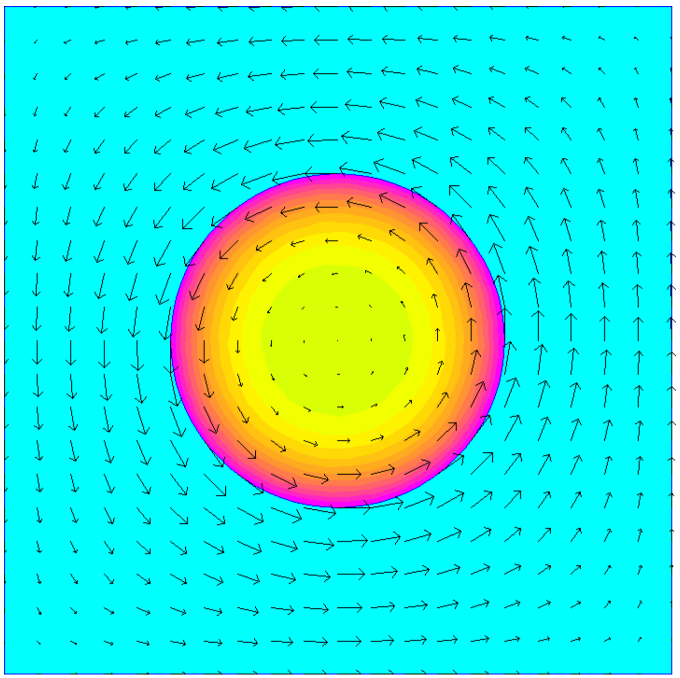


Current density in a winding exposed to an AC magnetic field

Skin and Proximity Loss Effects

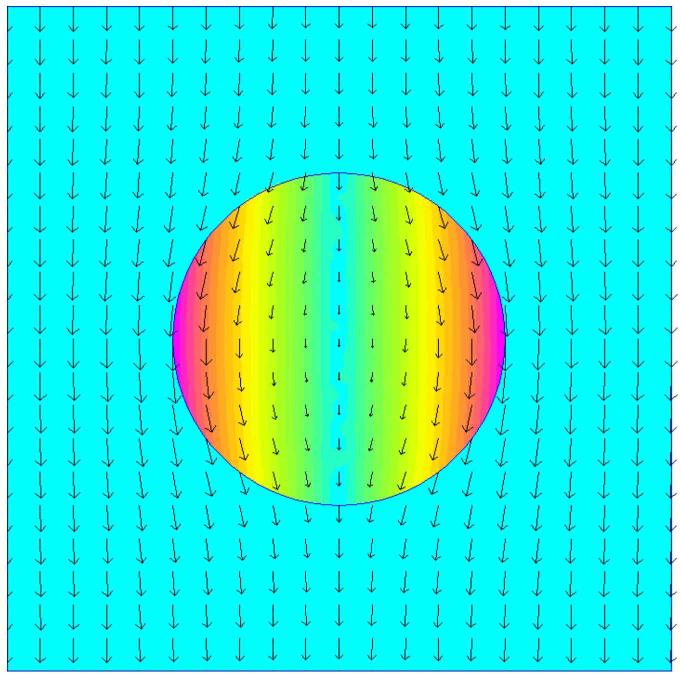
Skin Effect:

- Net current flows in conductor
- Current pushed towards surface of conductor by field due to current



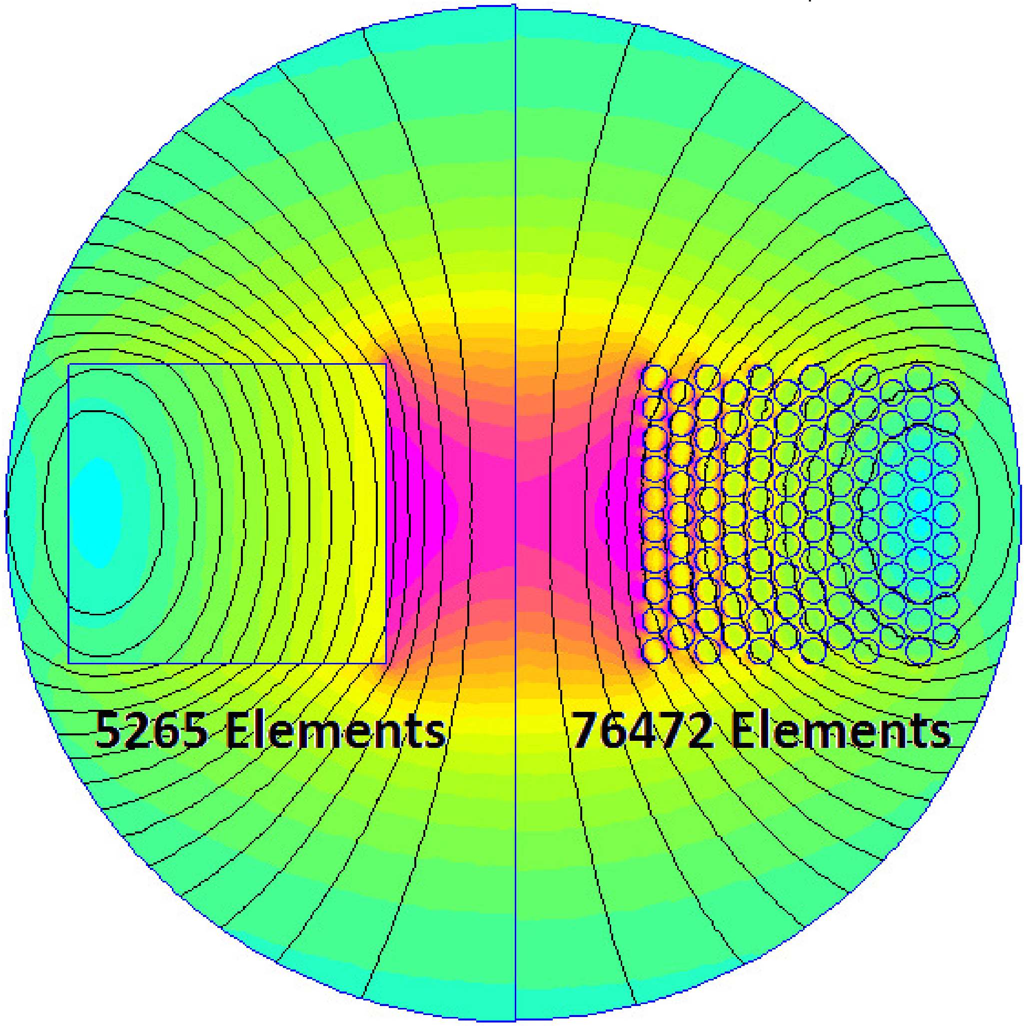
Proximity Effect:

- No net current in conductor
- Currents induced by externally applied field

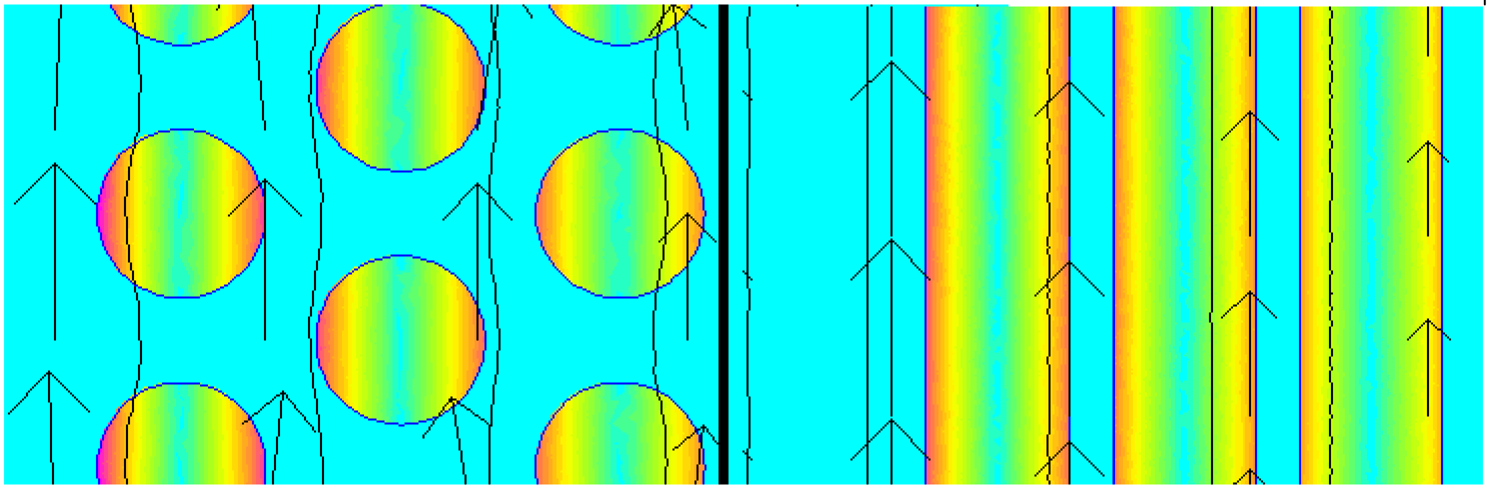


Motivation

- *Lots* of elements needed to explicitly represent each wire in a coil
- Desire a continuum method that lets us calculate accurate Proximity and Skin Effect losses
- Proximity effect captured by a complex-valued bulk magnetic permeability
- Skin effect captured by a complex-valued bulk conductivity
- Concentrating on the case of Hexagonal Packing



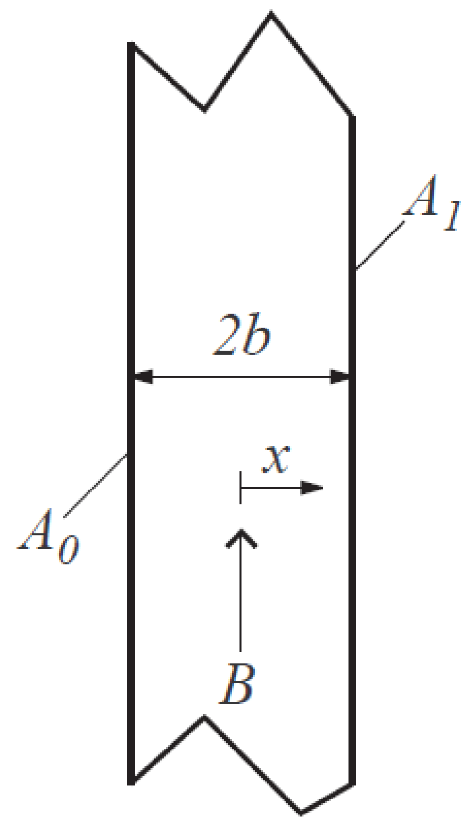
Equivalent Foil Model



- Classical approach: Pretend winding is a set of foils/bars
  - Can be solved analytically for equivalent permeability, conductivity
  - Rigorously shows how discrete conductors can be represented as a continuum
  - Can pick foil properties that give an OK agreement to wire winging
- Here, foil solution used as an approx form for fitting FEA results

Equivalent Foil Problem

Decompose problem into 3 sub-problems:



1. Impedance to flux with no bulk flux linkage to the coil (avg. A over foil=0)

$$A_{xx} = j\omega\sigma_f\mu_o A \text{ subject to } A(\pm b) = \pm\frac{A_1 - A_0}{2}$$

2. Resistive Losses (and some reactive power due to the local field)

$$A_{xx} = \sigma_f\left(j\omega\mu_o A + \mu_o \Delta V_r\right) \text{ subject to } A(\pm b) = 0$$

3. Flux linkage to the ambient magnetic field

$$A_{xx} = \sigma_f\left(j\omega\mu_o A + \mu_o \Delta V_i\right); \quad A(\pm b) = \frac{A_1 + A_0}{2}$$

Approximate Form for Curve Fit

Analytical solution from foil with gap ϵ between foils:

$$\mu_{eff} = \left(\frac{b}{b+\epsilon}\right)\mu_{fd} + \left(\frac{\epsilon}{b+\epsilon}\right)\mu_o$$

where

$$\mu_{fd} = \mu_o \frac{\tanh\sqrt{j\omega\sigma_f\mu_o b^2}}{\sqrt{j\omega\sigma_f\mu_o b^2}}$$

$$\rho_r = \left(\left(\frac{\mu_o}{\sigma_f\mu_{fd}}\right)\left(\frac{b+\epsilon}{b}\right) + j\omega\mu_o\epsilon(b+\epsilon)\right)$$

b/(b+ ϵ) is foil's fill factor

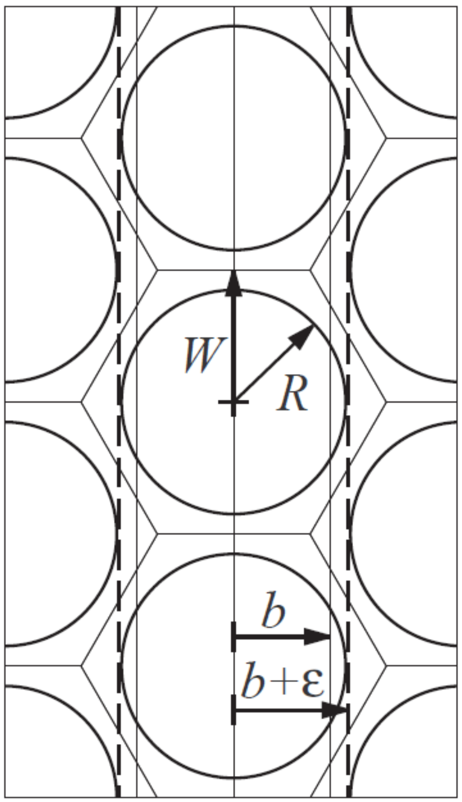
Approximate form for fitting to FEA data:

$$\mu_{eff} = (1-c_2)\mu_o + c_2\mu_o\frac{\tanh\sqrt{jc_1\Omega}}{\sqrt{jc_1\Omega}}$$

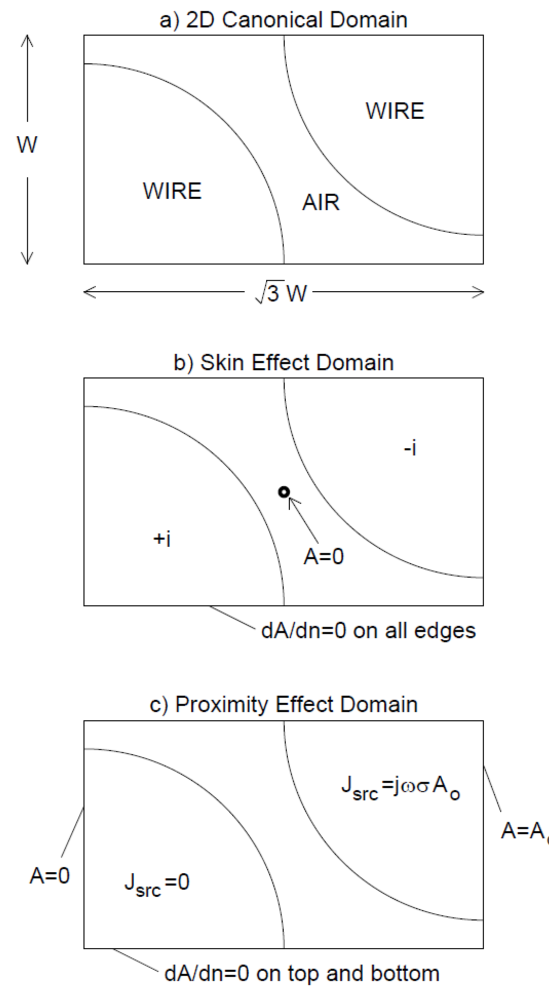
$$\rho_r = \left(\frac{1}{\sigma\,\mathrm{fill}}\right)\left(\frac{\sqrt{jc_3\Omega}}{\tanh\sqrt{jc_3\Omega}} + jc_4\Omega\right)$$

Where c1, c2, c3, c4 are functions of copper fill factor

FEA Skin and Proximity Domains



- For foil, easy to tell where to draw the boundary conditions.
- For hex winding, no clear edges where foil-like BCs can be defined
- Instead, define slightly different domains that yield the same computational results.

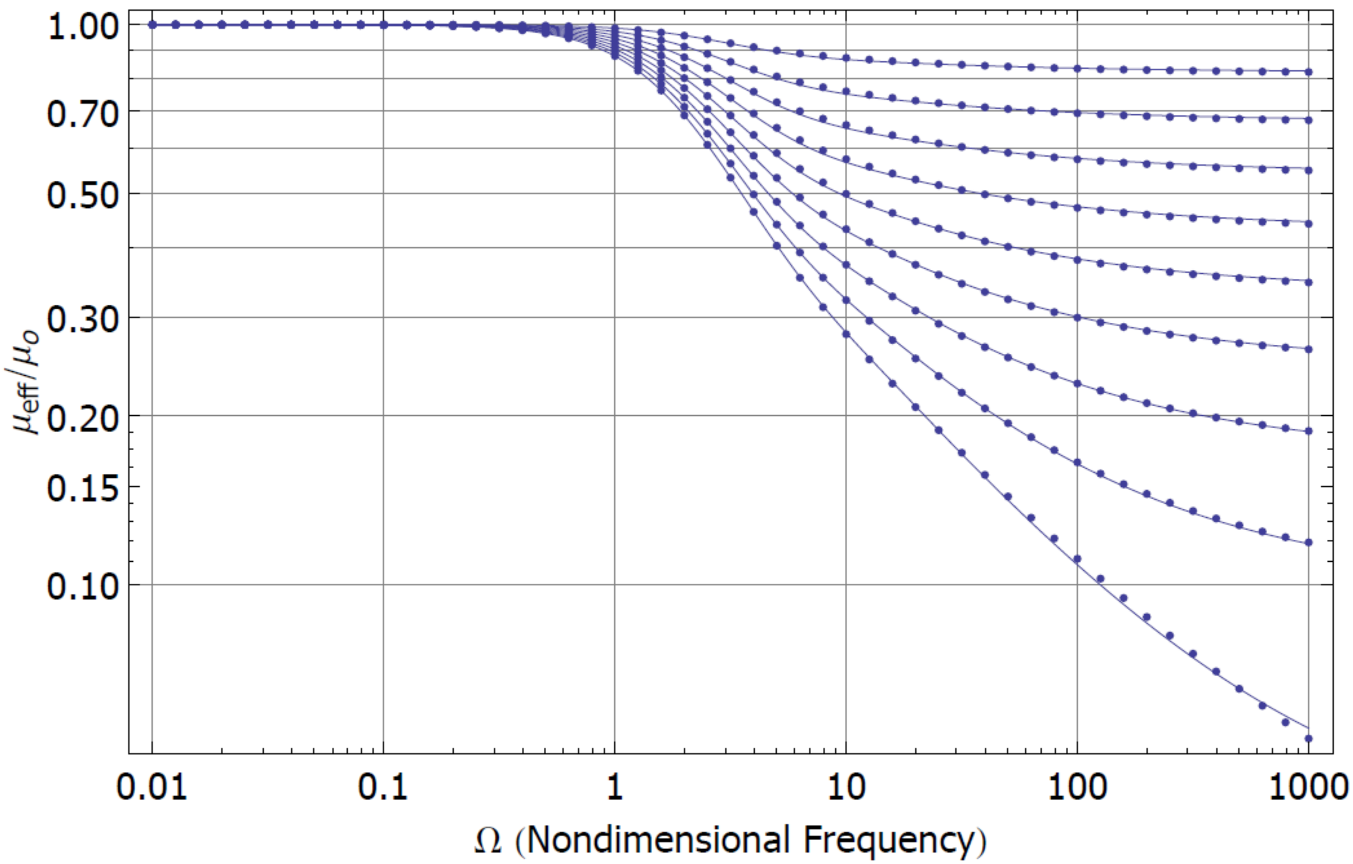


Parameter Fitting Approach

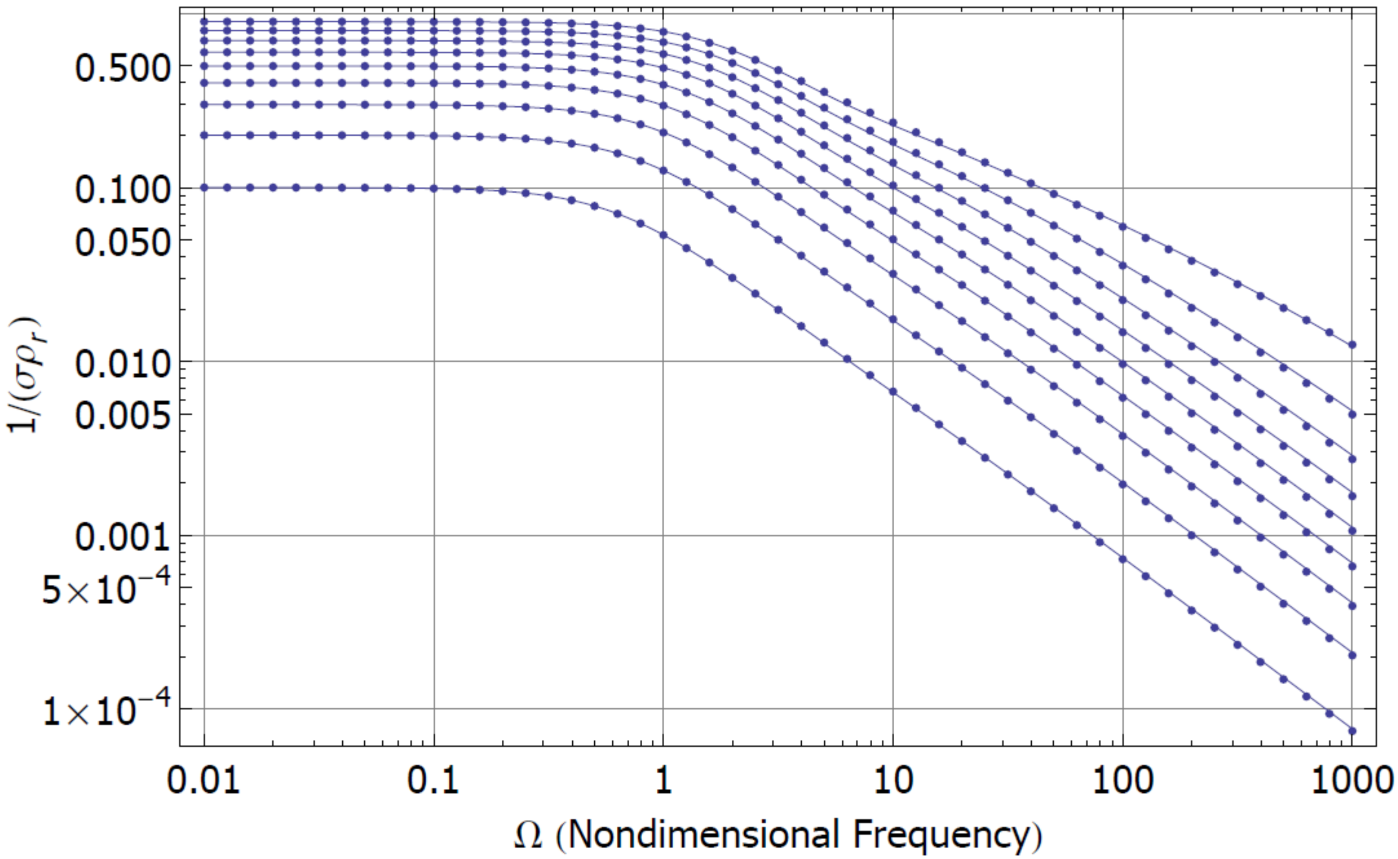
- Define c_2 in terms of c_1 to exactly satisfy an analytical expression for low-frequency effective permeability
- Define c_4 in terms of c_3 to exactly satisfy an expression for low-frequency / low fill factor effective resistivity
- Assume cubic polynomial forms for c_1 and c_3
- Choose parameters to minimize the normalized RMS Error:

$$\text{Error} = \left[\sum_{1}^{n} \frac{1}{n} \left| \frac{z_n - \hat{z}_n}{\hat{z}_n} \right|^2 \right]^{\frac{1}{2}}$$

Fit to Effective Permeability

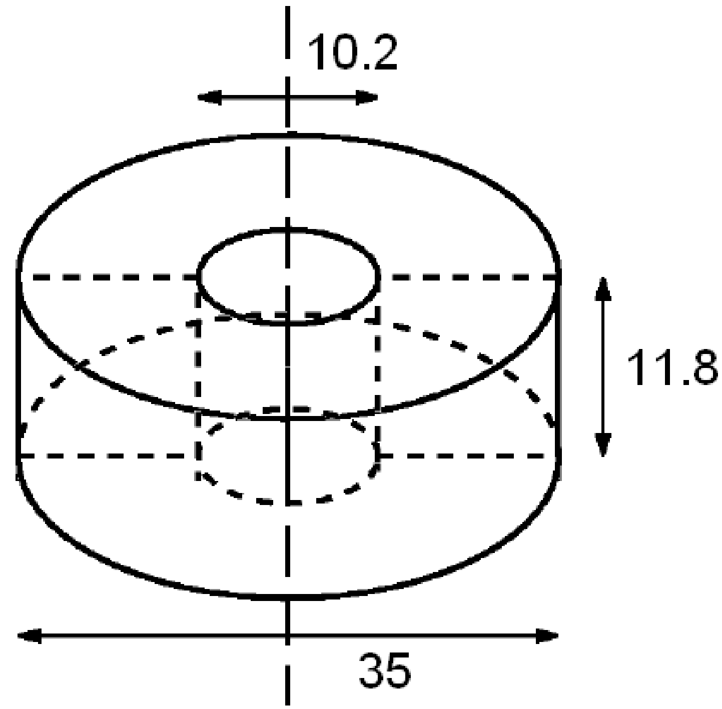


Fit to Effective Resistivity

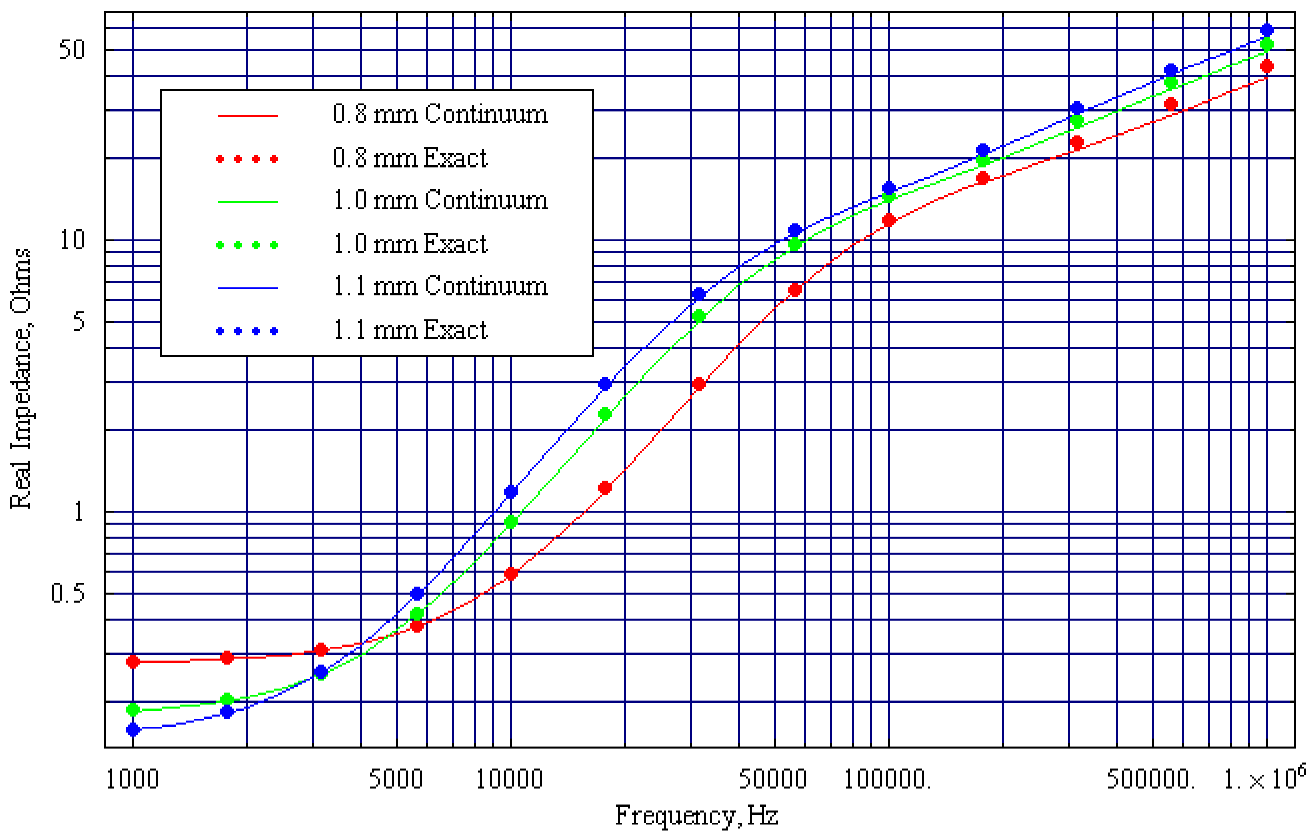


Example

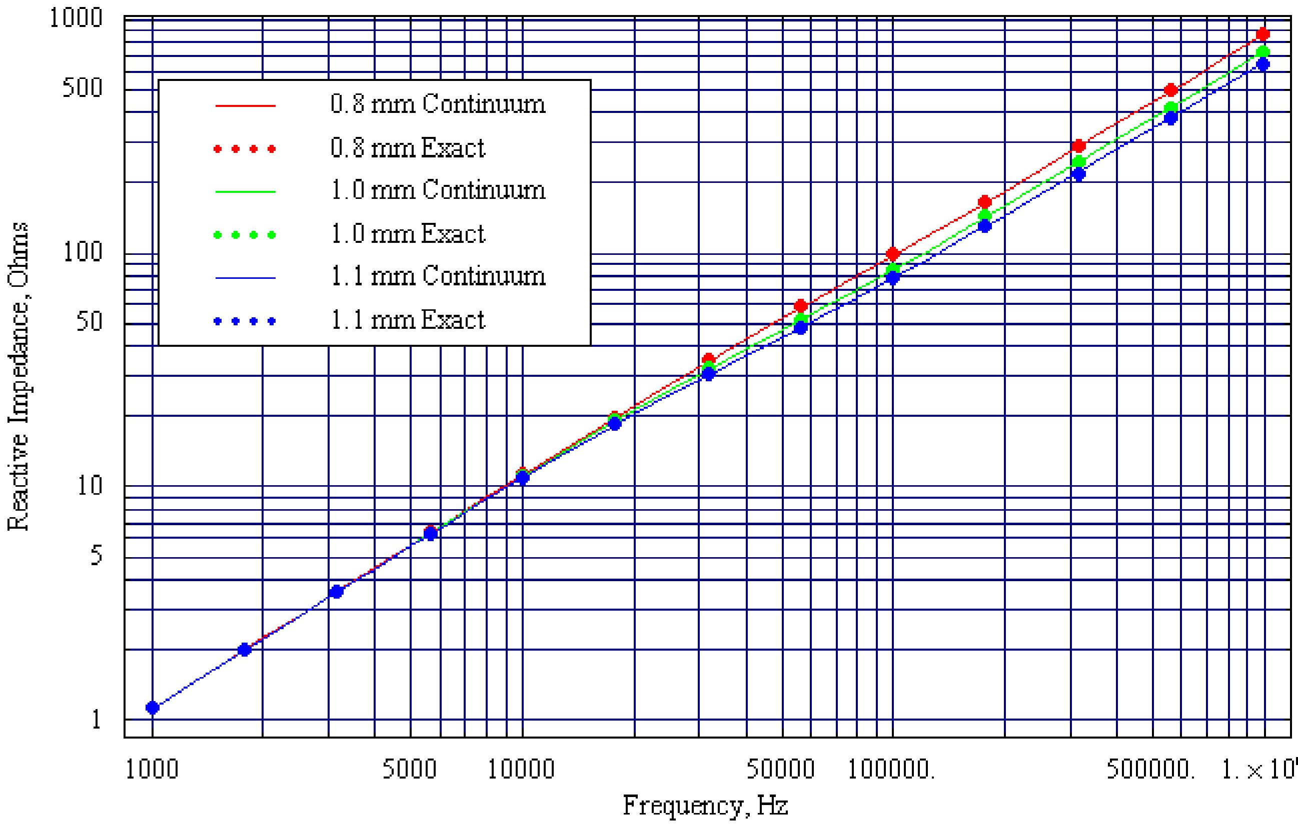
- Coil with copper wire
- ID = 10.2 mm
- OD = 35 mm
- Axial Length = 11.8 mm
- 114 turns
- Consider wire diameters of 0.8, 1.0, 1.1 mm



Exact and Approx Resistance



Exact and Approx Inductance



Conclusions

- Approximate but closed-form expressions made for permeability and conductivity that accurately represent Skin, Prox effects in hexagonally wound coils
- Expressions based on a form suggested by the analytical solution to equivalent foils
- Parameters identified via curve fit to a large number of finite element results
- Approach is now implemented in FEMM 4.2 and “automatically” applied to every wound coil.