Improved Continuum Skin and Proximity Effect Model for Hexagonally Packed Wires

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Induced currents

- When conductors are exposed to a time-varying magnetic field, currents are induced.
- Losses that occur when conducting wires are exposed to a magnetic field.
- Known as “Skin Effect” and “Proximity Effect”.

Current density in a winding exposed to an AC magnetic field.
Skin and Proximity Loss Effects

Skin Effect:
- Net current flows in conductor
- Current pushed towards surface of conductor by field due to current

Proximity Effect:
- No net current in conductor
- Currents induced by externally applied field
Motivation

- *Lots* of elements needed to explicitly represent each wire in a coil
- Desire a continuum method that lets us calculate accurate Proximity and Skin Effect losses
- Proximity effect captured by a complex-valued bulk magnetic permeability
- Skin effect captured by a complex-valued bulk conductivity
- Concentrating on the case of Hexagonal Packing
Equivalent Foil Model

- Classical approach: Pretend winding is a set of foils/bars
  - Can be solved analytically for equivalent permeability, conductivity
  - Rigorously shows how discrete conductors can be represented as a continuum
    - Can pick foil properties that give an OK agreement to wire winging
- Here, foil solution used as an approx form for fitting FEA results
Equivalent Foil Problem

Decompose problem into 3 sub-problems:

1. Impedance to flux with no bulk flux linkage to the coil (avg. A over foil=0)

\[ A_{xx} = j \omega \sigma_f \mu_0 A \text{ subject to } A(\pm b) = \pm \frac{A_1 - A_0}{2} \]

2. Resistive Losses (and some reactive power due to the local field)

\[ A_{xx} = \sigma_f \left( j \omega \mu_0 A + \mu_0 \Delta V_r \right) \text{ subject to } A(\pm b) = 0 \]

3. Flux linkage to the ambient magnetic field

\[ A_{xx} = \sigma_f \left( j \omega \mu_0 A + \mu_0 \Delta V_i \right); \quad A(\pm b) = \frac{A_1 + A_0}{2} \]
Approximate Form for Curve Fit

Analytical solution from foil with gap $\varepsilon$ between foils:

$$\mu_{eff} = \left(\frac{b}{b+\varepsilon}\right)\mu_{fd} + \left(\frac{\varepsilon}{b+\varepsilon}\right)\mu_o$$

where

$$\mu_{fd} = \mu_o \frac{\tanh \sqrt{j\omega\sigma_f \mu_o b^2}}{\sqrt{j\omega\sigma_f \mu_o b^2}}$$

$$\rho_r = \left(\frac{\mu_o}{\sigma_f \mu_{fd}}\right) \left(\frac{b+\varepsilon}{b}\right) + j\omega\mu_o \varepsilon (b+\varepsilon)$$

Approximate form for fitting to FEA data:

$$\mu_{eff} = (1-c_2)\mu_o + c_2\mu_o \frac{\tanh \sqrt{j\omega c_1 \Omega}}{\sqrt{j\omega c_1 \Omega}}$$

$$\rho_r = \left(\frac{1}{\sigma_{\text{fill}}}\right) \left(\frac{\sqrt{j\omega c_3 \Omega}}{\tanh \sqrt{j\omega c_3 \Omega}} + j\omega c_4 \Omega\right)$$

$b/(b+\varepsilon)$ is foil’s fill factor

Where $c_1, c_2, c_3, c_4$ are functions of copper fill factor
FEA Skin and Proximity Domains

- For foil, easy to tell where to draw the boundary conditions.
- For hex winding, no clear edges where foil-like BCs can be defined
- Instead, define slightly different domains that yield the same computational results.
Parameter Fitting Approach

• Define $c_2$ in terms of $c_1$ to exactly satisfy an analytical expression for low-frequency effective permeability.

• Define $c_4$ in terms of $c_3$ to exactly satisfy an expression for low-frequency / low fill factor effective resistivity.

• Assume cubic polynomial forms for $c_1$ and $c_3$.

• Choose parameters to minimize the normalized RMS Error:

$$\text{Error} = \left[ \sum_{1}^{n} \frac{1}{n} \left| \frac{z_n - \hat{z}_n}{\hat{z}_n} \right|^2 \right]^{\frac{1}{2}}$$
Fit to Effective Permeability
Fit to Effective Resistivity
Example

- Coil with copper wire
- ID = 10.2 mm
- OD = 35 mm
- Axial Length = 11.8 mm
- 114 turns
- Consider wire diameters of 0.8, 1.0, 1.1 mm
Exact and Approx Resistance

![Graph showing the relationship between Real Impedance (Ohms) and Frequency (Hz) for different continuum and exact approximations at various mm sizes. The graph includes data points and trend lines for 0.8 mm, 1.0 mm, and 1.1 mm continua and exact approximations.]
Exact and Approx Inductance

![Graph showing the relationship between frequency and reactive impedance for different Continuum and Exact cases.]

- 0.8 mm Continuum
- 0.8 mm Exact
- 1.0 mm Continuum
- 1.0 mm Exact
- 1.1 mm Continuum
- 1.1 mm Exact
Conclusions

- Approximate but closed-form expressions made for permeability and conductivity that accurately represent Skin, Prox effects in hexagonally wound coils
- Expressions based on a form suggested by the analytical solution to equivalent foils
- Parameters identified via curve fit to a large number of finite element results
- Approach is now implemented in FEMM 4.2 and “automatically” applied to every wound coil.