Title: Influence of actuator geometry on rotating losses in heteropolar magnetic bearings

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ABSTRACT

Recently, work on rotating losses in magnetic bearings has focused mainly on the measurement of rotating losses, and on the creation of models that attempt to reproduce these results. Though there has been some success on both counts, there has been less emphasis on interpreting what these results mean in terms of practical guidelines for the design of low-loss bearings. The present work reformulates a previously developed analytical model of rotating loss so that the effects shaft speed and pole count on rotating losses can be more easily identified. Conclusions drawn from this formulation are then compared to previously reported experimental data.

INTRODUCTION

Most recent work on rotating losses in heteropolar magnetic bearings generally has one of two aims: either accurate measurement these losses, or development of analytical or computational models for the losses. Losses have been measured via run-down tests (Kasarda, 1997; Mizuno and Higuchi, 1994). Alternatively, losses can be deduced thermally (Stephens, 1996). Models of the losses in heteropolar bearings have been derived for the case of laminated rotors (Meeker and Maslen, 1998), and for solid rotors (Ahrens and Kučera, 1996). Both of these works approach the problem via a Fourier analysis of the magnetic field, as originally suggested in (Matsumura and Hatake, 1992). These models address eddy current losses but have not yet been extended to include hysteresis or saturation effects. However, these previous works do not explore the implications of these results on the design of lower-loss heteropolar bearings. The main design question addressed by these works is the merits of NSNS biasing versus NNSS biasing. However, other aspects of geometry also have important effects on the amount of rotating loss: number of poles on the stator; thickness of the journal iron; and the width of the poles. The aim of the present work is to reformulate the loss model developed in (Meeker and

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Maslen, 1998) so that the effects of the bearing geometry are clearly in evidence. Conclusions drawn from this model are compared with experimental results from (Kasarda, 1997).

In outline, the model development presented previously in (Meeker and Maslen, 1997) is reviewed. Next, this model is reformulated more clearly expose the significant parameters controlling the losses. Then, the effect of shaft speed is examined. A nondimensionalized formulation needed for exploration of geometric effects is developed, and the effect of pole count is explored. These findings are compared to data reported in (Kasarda, 1997). The paper concludes with summary remarks and suggestions for natural extensions.

MODEL REVIEW

For rotationally-induced eddy currents in laminated heteropolar bearings, a thin-plate model can be employed that is in some ways similar to the eddy current models used for losses in transformer cores (Stoll, 1974). Since the laminations are thin in comparison to the other dimensions of the journal, simplifications can be made which permit an analytical solution for the field inside the journal laminations in terms of the field at the surface of the journal. Rotating losses inside the journal are inferred from the field at its surface. The reader is directed to (Meeker and Maslen, 1998) for a full derivation of the present loss model; only the relevant results will be presented here.

The model assumes that the journal iron is magnetically linear, has constant, isotropic permeability μ and conductivity σ, and has negligible hysteresis. The thickness of the rotor laminations is represented by d. The coordinates θ and r denote tangential and radial position relative to the center of the journal in a stator-fixed reference frame.

Since field in the rotor is 2π periodic in the θ coordinate, the magnetic field solution consists of harmonics in θ. A phasor representation (Hoole, 1989) can be adopted where average flux density across the thickness of a lamination, $\bar{B}$, is:

$$\bar{B}(r,\theta) \equiv \mathrm{Re}\left[\sum_{n=0}^{\infty} \bar{b}_n(r)\, e^{jn\theta}\right] = \mathrm{Re}\left[\sum_{n=0}^{\infty} \bar{b}_n(r)\,(\cos n\theta + j\, sin\, n\theta)\right] \tag{1}$$

where $\bar{b}_n$ is a complex number denoting the magnitude and phase of the n^{th} harmonic component of $\bar{B}$. Since the system is assumed to be linear, each harmonic can be considered separately and the results for all harmonics superimposed to yield a complete solution. In the same way, magnetic scalar potential, Ω, can be represented as a phasor transform.

The main result of (Meeker and Maslen, 1998) is that the effects of eddy currents inside the journal are represented solely by a boundary condition for each harmonic that relates scalar potential applied to the surface of the journal to its normal derivative:

$$\frac{\partial \Omega_n}{\partial r} = \left(\frac{\mu n}{\mu_o r_o}\right)\left\{\frac{\tanh[(1+j)\frac{d}{2\delta_n}]}{(1+j)\frac{d}{2\delta_n}}\right\}\tanh\left[\tfrac{n}{r_o}(r_o - r_i)\right]\Omega_n \tag{2}$$

Where δ_n is the skin depth for the n^{th} harmonic component:

$$\delta_n = \sqrt{\frac{2}{n\omega\sigma\mu}} \tag{3}$$

This boundary condition can then be used in combination with a BEM or FEM model of the bearing to solve for the scalar potential distribution on the surface of the rotor.

Once the distribution of potential is known on the surface of the rotor, the power loss associated with each harmonic component can be computed by integrating the power losses implied by the flux distribution inside the journal. For each harmonic component, the loss for the entire journal is:

$$P_n = |\Omega_{n,o}|^2 \left\{ \frac{2\pi\, n\, l}{\sigma \delta_n\, d} \right\} \tanh \left[\tfrac{n}{r_o}(r_o - r_i) \right] \left\{ \frac{\sinh \frac{d}{\delta_n} - \sin \frac{d}{\delta_n}}{\cosh \frac{d}{\delta_n} + \cos \frac{d}{\delta_n}} \right\} \tag{4}$$

where l is the axial length of the journal and $\Omega_{n,o}$ denotes the magnitude of the n^{th} harmonic of scalar potential at the surface of the rotor.

REFORMULATION OF LOSS EQUATION

In short, there are two main results of the thin plate eddy current model. The first is eq. (2), a tool for relating the potential at the rotor surface to the flux that is flowing normal to the rotor surface. If this boundary condition is used, the flux in the journal need not be found explicitly, because an analytical solution for the flux in the journal is implied by this boundary condition. The second is Eq. (4), an equation for eddy current losses only in terms of potential at the rotor's surface, the material properties of the journal laminations, and the lamination thickness. Once the potential at the surface is determined, the total rotating eddy current losses can be determined by summing the loss components for each harmonic:

$$P_{total} = \sum_{n=1}^{\infty} P_n \tag{5}$$

Although these results are useful for a computational study of bearing losses, their utility is somewhat limited. In the form of (2) and (4), nothing in particular is evident about the effects of varying bearing geometry (except, perhaps, the trivial result that losses can be reduced by thinner laminations). However, these equations can be reformulated in a way that illuminates the effects of various geometric parameters on rotating losses.

A crucial insight comes from several researchers (Meeker and Maslen, 1998; Matsumura and Hatake, 1992): in the case of *laminated* rotors, the flux profile in the air gap varies almost negligibly over a very wide range of speeds. Even though the reluctance of the rotor rises due to the effects of rotationally-induced eddy currents, journal reluctance remains small compared to the highly reluctant air gap. The result is a relatively constant flux density profile over a wide range of speeds. If the expression for loss can be rewritten in terms of flux density at the rotor surface, rather than potential, the magnetostatic flux distribution (or some idealized flux distribution) can be used to estimate the losses, providing a closed-form approximation for the losses, instead of the previous computational/analytical form. In this manner, the effects of different geometric parameters can be explored directly.

The goal is to solve for $\Omega_{n,o}$ in terms of $\bar{b}_{n,o}$, which denotes the n^{th} harmonic component of the average flux density directed normal to the rotor at the surface of journal. The loss will be in terms of flux at the surface of the journal. Since $\bar{b}_{n,o}$ is the flux density in the air at the journal surface, it can be written in terms of the scalar potential in the air gap as:

$$\bar{b}_{n,o} = -\mu_o \frac{\partial \Omega_n}{\partial r} \tag{6}$$

Substituting (6) into boundary condition (2) relates $\Omega_{n,o}$ to $\bar{b}_{n,o}$:

$$\bar{b}_{n,o} = -\left(\frac{\mu n}{r_o}\right) \left\{ \frac{\tanh[(1+j)\frac{d}{2\delta_n}]}{(1+j)\frac{d}{2\delta_n}} \right\} \tanh\left[\tfrac{n}{r_o}(r_o - r_i)\right] \Omega_n \tag{7}$$

Taking the magnitude of both sides of (7) gives, after considerable simplification:

$$\left|\bar{b}_{n,o}\right|^2 = |\Omega_{n,o}|^2 \left\{ \frac{\mu n}{r_o} \tanh\left[\tfrac{n}{r_o}(r_o - r_i)\right] \right\}^2 \left\{ \frac{2\delta_n^2}{d^2} \right\} \left\{ \frac{\cosh\frac{d}{\delta_n} - \cos\frac{d}{\delta_n}}{\cosh\frac{d}{\delta_n} + \cos\frac{d}{\delta_n}} \right\} \tag{8}$$

Solving (8) for $|\Omega_{n,o}|^2$ and substituting into (4) gives

$$P_n = \left|\bar{b}_{n,o}\right|^2 \left\{ \frac{\pi r_o^2\, l\, d}{n \mu^2 \sigma \delta_n^3} \right\} \left\{ \frac{\sinh\frac{d}{\delta_n} - \sin\frac{d}{\delta_n}}{\cosh\frac{d}{\delta_n} - \cos\frac{d}{\delta_n}} \right\} \coth\left[\tfrac{n}{r_o}(r_o - r_i)\right] \tag{9}$$

Equation (9) can then be rearranged as the product of a term representing effective loss per unit volume, $P_{e,n}$, and an effective journal volume, $V_{e,n}$:

$$P_n = P_{e,n}\, V_{e,n} \tag{10}$$

where

$$P_{e,n} = \frac{1}{2} \left\{ \frac{|\bar{b}_{n,o}|^2\, d}{\mu^2 \sigma \delta_n^3} \right\} \left\{ \frac{\sinh\frac{d}{\delta_n} - \sin\frac{d}{\delta_n}}{\cosh\frac{d}{\delta_n} - \cos\frac{d}{\delta_n}} \right\} \tag{11}$$

$$V_{e,n} = 2\pi r_o\, l\, (r_o - r_i) \left\{ \frac{\coth[n\, w]}{n\, w} \right\} \tag{12}$$

and w is the non-dimensional journal thickness ("journal fraction"):

$$w \equiv \frac{r_o - r_i}{r_o} \tag{13}$$

Two interesting points are directly evident from these forms. First, (11) has exactly the same form as the classical expression for loss per unit volume. Second, (12) is just the volume of the unrolled journal times a correction factor based on the thickness of the journal and the number of the harmonic in consideration. As n increases, the effective volume becomes smaller because most of the flux stays close to the surface of the journal.

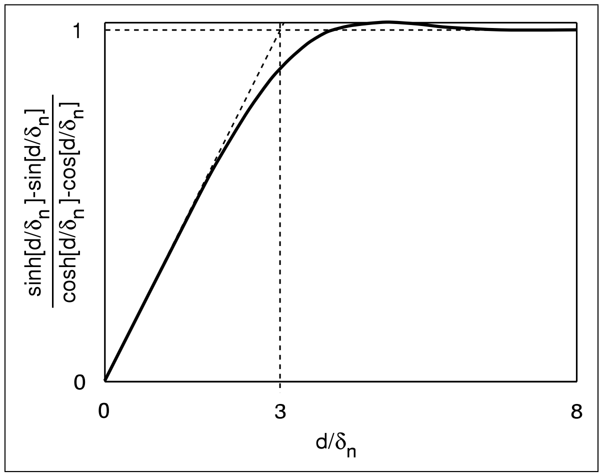

Figure 1: A component of $P_{e,n}$ versus d/δ_n

EFFECT OF SHAFT SPEED

Since shaft speed, ω, appears only as a component of δ_n in $P_{e,n}$, its effects can be considered by examining this term alone. Previous works have asserted that rotating eddy current losses should increase proportionally with ω^2 (Kasarda, 1997; Matsumura and Hatake, 1992). This dependence arises by considering only the first term in the Taylor expansion of the classic loss/area equation about $\omega = 0$. If (11) is also expanded about $\omega = 0$, the result is:

$$P_{e,n} \approx \frac{1}{6}\left\{\frac{|\bar{b}_{n,o}|^2\, d^2}{\mu^2\sigma\delta_n^4}\right\} = \frac{1}{24}|\bar{b}_{n,o}|^2\sigma\, n^2\omega^2 d^2 \tag{14}$$

The ω^2 loss dependence is shown in the present work in the case where d/δ_n is small.

However, this approximation must be judiciously applied for an accurate result. Consider the second bracketed term in (11), denoted for ease of notation as u:

$$u \equiv \frac{\sinh\frac{d}{\delta_n} - \sin\frac{d}{\delta_n}}{\cosh\frac{d}{\delta_n} - \cos\frac{d}{\delta_n}} \tag{15}$$

The u term is plotted in Figure 1. The low-frequency approximation is obtained by approximating u as its low-frequency asymptote,

$$u \approx \tfrac{1}{3}d/\delta_n \tag{16}$$

However, u has a high-frequency asymptote of $u = 1$. This asymptote leads to a high-frequency approximation of $P_{e,n}$ as:

$$P_{e,n} \approx \frac{1}{2}\left\{\frac{|\bar{b}_{n,o}|^2 d}{\mu^2\sigma\delta_n^3}\right\} = \frac{|\bar{b}_{n,o}|^2 d}{4\sqrt{2}\mu^2\sigma}\left[n\omega\sigma\mu\right]^{\frac{3}{2}} \tag{17}$$

The dependence on ω is to the 3/2 power, rather than squared.

There is potentially a large difference between the two approximations. To decide whether a given case is best approximated as low-frequency or high-frequency, consider the intersection of the two asymptotes. These lines meet at $d/\delta_n = 3$ (that is, when the lamination thickness is three times the skin depth), which might be used as a dividing line between the two regimes.

To get a feel for where this line typically falls, consider a journal composed of typical 0.35 mm (14 mil) laminations. If an eight pole stator run in a NSNS biasing scheme is considered, the lowest numbered harmonic present will be $n = 4$. From (Meeker and Maslen, 1998), measured properties for a particular sample of 3% Silicon Iron are σ=7.46 MS/m and μ=3460 μ_o. In this case, solving for ω when $d/\delta_n = 3$ yields ω=10500 RPM – roughly on the dividing line between what would be considered low speed and high speed rotors.

Even for low-speed rotors, (14) cannot be used indiscriminately; substituting (14) for $P_{e,n}$ in (5) typically does not result in a convergent series. Even though the lower-numbered harmonic components of the loss may be well approximated with (14), the higher-numbered parts fall in the the small skin depth paradigm. For high speed rotors, it may be most accurate to assume that loss goes with $\omega^{3/2}$. This way, losses will be more accurate at high frequencies where the impact of the losses is more likely to be of concern. Low frequency losses will be over-predicted, but the relative magnitude of these errors will be small.

NONDIMENSIONALIZED LOSS

To examine the effects of specific geometric parameters, it is useful to nondimensionalize the losses. Since losses are mainly of concern at high speeds, it is assumed that the $P_{e,n}$ can be approximated by (17).

The analysis is simplified somewhat by considering only the idealized flux distributions of bearings with an even number of poles, neglecting leakage and fringing in the gap. In this case, specific magnitudes can be prescribed for each $\bar{b}_{m,o}$. For a bearing with p poles, each pole has a width of $2\pi F_p/p$ radians, where F_p is the fraction of the journal's surface covered by poles: $0 < F_p < 1$; a typical value of F_p is about 0.5. Under each pole, there is a uniform flux density of magnitude B_{bias} normal to the journal; between poles, no flux crosses the journal's surface. The idealized flux distributions for both the NSNS and NNSS biasing schemes are plotted in Figure 2. With these simple flux distributions, the sequence of $\bar{b}_{n,o}$ can be represented analytically by computing the phasor transformations of the above profiles. The result of the phasor transformation is that all $\bar{b}_{n,o}$ are zero except for n where:

$$n = \frac{(2m-1)p}{2\,q} \quad \text{for} \quad m = 1, 2, \ldots \tag{18}$$

where $q = 1$ for NSNS and $q = 2$ for NNSS. The nonzero $\bar{b}_{n,o}$ are:

$$\bar{b}_{m,o} = \left\{ \frac{-j4\,q\,B_{bias}}{(2m-1)\pi} \right\} \sin\left[\frac{(2m-1)\pi}{2\,q} \right] \sin\left[\frac{(2m-1)\pi}{2\,q} F_p \right] \tag{19}$$

Equations (18) and (19) show that changing the number of poles in the bearing doesn't

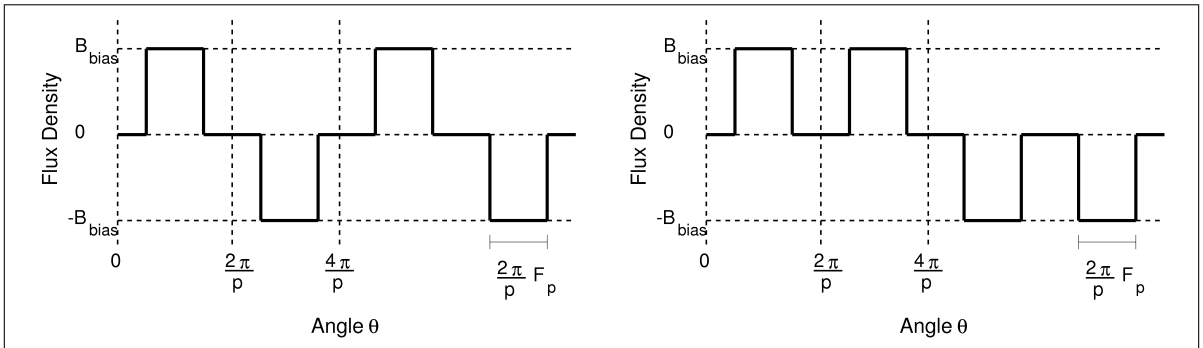

Figure 2: Ideal flux distributions

change the values of the sequence of non-zero coefficients; it merely shifts the locations where they occur in the sequence of n's.

To compare losses produced by various pole widths in a meaningful way, these losses should be compared for bearings with the same total pole area (so the same net amount of flux is going through the journal, and the load capacity remains constant). The total pole area,a_{iron}, is defined as:

$$a_{iron} = (2\pi r_o l) F_p \tag{20}$$

Solving for l yields:

$$l = \frac{a_{iron}}{2\pi r_o F_p} \tag{21}$$

Since r_i is usually specified by the shaft diameter, and r_o is chosen later, it is useful to substitute for r_o in terms of w and r_i:

$$r_o = \frac{r_i}{1 - w} \tag{22}$$

Substituting (19) into (17), the high-speed approximation for $P_{e,n}$, yields:

$$P_{e,m} \approx \left\{ \frac{B_{bias}^2 d}{4\sqrt{2}\mu^2\sigma} \left[\omega\sigma\mu\right]^{\frac{3}{2}} \right\} \bar{P}_{e,m} \tag{23}$$

where $\bar{P}_{e,m}$ is the nondimensional loss for each harmonic:

$$\bar{P}_{e,m} = \left\{ \frac{4\, p^{3/2}}{\pi^2} \right\} \left[\frac{2}{(2m-1)q} \right]^{1/2} \sin^2 \left[\frac{(2m-1)\pi}{2\, q} F_p \right] \tag{24}$$

Substituting (21) and (22) into (12) yields:

$$V_{e,n} = (a_{iron} r_i)\, \bar{V}_{e,m} \tag{25}$$

where the nondimensional journal volume, $\bar{V}_{e,m}$ is:

$$\bar{V}_{e,m} = \left\{ \frac{2\, q}{p\, F_p\, (2m-1)(1-w)} \right\} \coth \left[\frac{(2m-1) p\, w}{2\, q} \right] \tag{26}$$

The total non-dimensional loss, Q, for the bearing can then be obtained by summing the contributions from each harmonic:

$$Q = \sum_{m=1}^{\infty} \bar{P}_{e,m} \bar{V}_{e,m} \tag{27}$$

Note that Q is independent of speed, material properties, and lamination thickness; it is a function of w, F_p, and p, as well as the choice of biasing configuration, q. In addition, Q is bounded by a series proportional to $m^{-3/2}$, guaranteeing that Q converges.

EFFECT OF POLE COUNT

From (24), the loss per volume increases with $p^{3/2}$. Therefore, increasing the number of poles has the same effect on the loss per volume term as increasing the shaft speed, as might be expected. However, the increase in the loss per unit volume is largely offset by a decrease in effective volume. Eq. (26) shows that the effective volume goes with (at worst) $1/p$. At higher n, the flux tends to stay closer to the surface of the journal. Multiplying the contributions from $\bar{P}_{e,m}$ and $\bar{V}_{e,m}$ yields a $\sqrt{p}$ dependence of loss on pole count.

Scaling by $\sqrt{p}$, however, represents a worst-case increase of losses with increasing pole number. Due to the cotangent component of $\bar{V}_{e,m}$, the lower-numbered components of the loss can scale at less than $\sqrt{p}$. If $(p\,w)/(2\,q)$ is small (when the journal is thin relative to the number of poles), flux becomes significantly more concentrated than it would be for either a thicker journal or a greater number of poles case. The effective volume rises, and loss is higher. Increasing the number of poles while maintaining the same journal iron thickness can pull the the lowest-numbered components out of this high-loss region, resulting in the less than $\sqrt{p}$ increase in losses.

COMPARISON TO EXPERIMENTAL DATA

A previous experimental study (Kasarda, 1997) measured the rotating losses in magnetic bearings with a range of configurations. This work was based on measuring the rundown rate of the suspended shaft under the combined influences of windage, hysteresis, and eddy current losses. (Mechanical friction losses in the shaft were minimized by using a very short, rigid shaft which should exhibit very little bending at the tested speeds.) It is important to note that, as with virtually any power loss measurement it was impossible for Kasarda to measure eddy current losses directly. Instead, the losses were inferred from rotor speed versus time trajectories on the basis of a model which attempted to predict the influence of windage, eddy currents, and hysteresis on this trajectory. Therefore, the eddy current losses reported in (Kasarda, 1997; Minuzo and Higuchi, 1994) are substantially colored by the model used to extract them.

Of particular interest in this study is the comparison between the losses in eight and sixteen pole stator bearings. The stators were designed so that the total pole area was the same for both designs. Two different shafts were used in the testing so that results were obtained both with 0.38 mm (15 mil) radial air gaps and with 0.76 mm (30 mil) radial air gaps. Both shafts had journals composed of 3% silicon iron with a lamination thickness

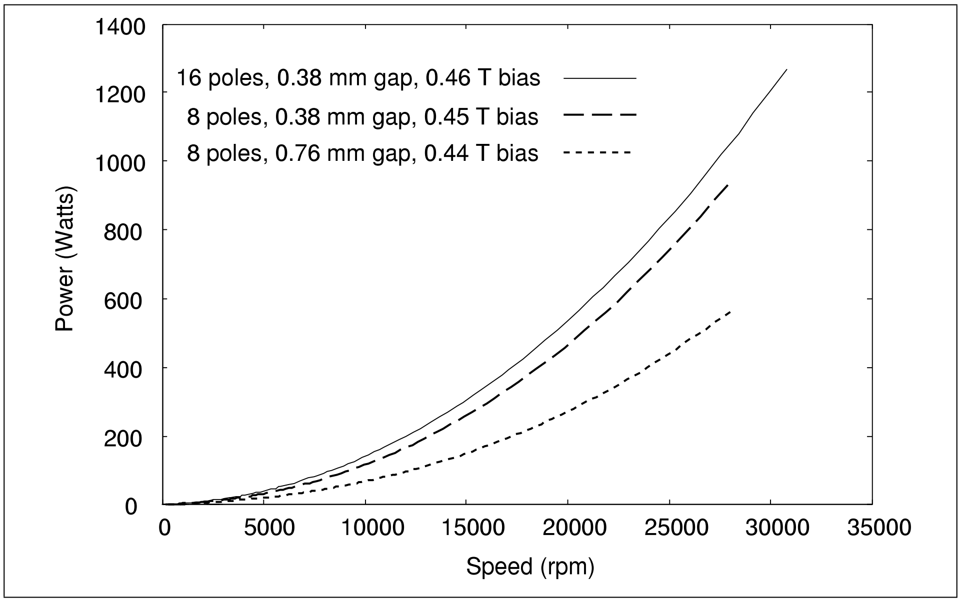

Figure 3: Measured losses for various pole numbers and air gap lengths

of 0.36 mm (14 mils). Tests were run at bias levels of 0.32 Tesla, 0.38 Tesla, 0.46 Tesla, and 0.54 Tesla. In all cases where results with similar air gaps and bias flux densities are compared, the eddy current losses appeared to be identical (within the experimental uncertainty) for the 8- and 16- pole stators. The data for the nominal 0.46 Tesla test do not agree exactly, but the reported bias for the 8- pole stator is 0.44 Tesla which should lead to about 10 percent lower losses, consistent with the experimental results. Examples of the experimentally determined losses for these cases are pictured in Figure 3, taken from data presented in (Kasarda, 1997).

To be consistent with the discussion above, which indicates that, if pole fraction F_p, axial length ℓ, and rotor radius r_i are all held constant, then the losses should be at most 40 percent greater for the 16 pole bearings than for the eight pole bearings. In the case of a change from 8 to 16 poles while keeping a constant air gap length, the increase in losses is very modest–less than a $p^{1/2}$ increase (the expected worst case rate of increase in the present analysis).

However, the experimental data shows that, for the same bias field density, the losses increase by nearly 100 percent when the number the air gap is decreased from 0.76 mm to 0.38 mm. This is not consistent with the present model, in which fringing effects are neglected (i.e. the same losses would be expected for both airgap lengths). Apparently, fringing effects around the pole tips may have a significant effect on the subsequent rotating losses.

It is also interesting to note that (Kasarda, 1997) used a model for the speed dependence of the eddy current losses which goes as ω^2 at low speeds but is mitigated by a "crowding" effect at higher speeds: the second term in a Taylor's series expansion of (11). A simple null hypothesis exploration of the importance of this second term on regressing the experimental data suggested that the term could not be significantly detected in the data. The importance of this is that it suggests that the data is dominated by "low" frequencies, which contradicts the present comment on the critical ratio of lamination thickness to skin depth. One possible explanation for this apparent contradiction is that the Taylor's series for (11) does not converge very quickly and, in fact, retaining just the first and second terms would imply an actual *reduction* of eddy current losses with speed increase beyond about 15,000 RPM in Kasarda's data: the model explored in the null hypothesis test was poor enough that its rejection would be expected even if the data was, in fact, substantially dominated by "high" frequency effects.

CONCLUSIONS

An analytical formulation was presented for predicting rotating losses in laminated heteropolar magnetic bearings. Several insights relevant to low-loss bearing designs were gleaned from this formulation:

- Losses become proportional to $\omega^{3/2}$ at high speed.
- Increasing the number of poles while maintaining a constant pole fraction scales the losses with $\sqrt{p}$ *in the worst case*.

Additional geometric parameters like pole fraction and journal width are also brought into evidence, although their effects on loss are not examined in this work.

Several significant discrepancies between previously reported experimental data and the present analysis were uncovered. Examining the models which underlie the regressions required to extract the experimental results suggests that some of these discrepancies may stem from flawed experimental models. However, a significant corroboration lies in the fact that the experimental work clearly suggests that the sensitivity of loss to the number of poles is relatively small.

In order to reveal some of the geometric sensitivities, the present work idealizes the field distribution around the journal as a square wave. The experimental data suggests that fringing at the edges of the air gap plays a significant role in mitigating eddy current losses. Therefore, an important extension of the present work would be to find a way to introduce this effect in a simple manner so that power loss optimal designs can take advantage of this effect as well.

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