FEA Results Postprocessing for a LRK Motor

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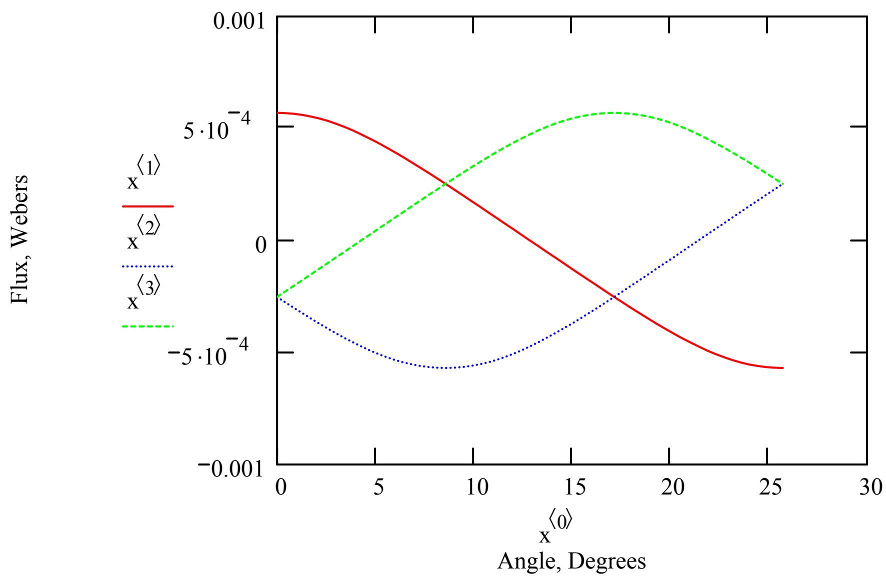
Introduction

A Lua script was created that evaluates the PM flux linkage for each phase at the cogging torque for a LRK motor at a number of different rotor positions. The purpose of this worksheet is to plot the results from the analyses and to differentiate the flux linkage to obtain back EMF per unit of rotor speed

Flux Linkage and Back EMF

Read in the FEA results from disk; $x :=$ C:\..\flux-results.txt



We need to differentiate the flux linkage with respect to position to get back-EMF. We could do this numerically, but a more elegant way to do this is to fit coefficients for the first few terms of a Fourier series representation of the data. The derivative of the Fourier series can then be taken analytically.

$$\text{fctn}(\theta) := \begin{pmatrix} \cos(7\cdot\theta\cdot\text{deg}) \\ \cos(3\cdot7\cdot\theta\cdot\text{deg}) \\ \cos(5\cdot7\cdot\theta\cdot\text{deg}) \\ \cos(7\cdot7\cdot\theta\cdot\text{deg}) \end{pmatrix}$$

These are the terms of the Fourier series that will be considered.

$$\mathrm{coeff} := \mathrm{linfit}\left(x^{\langle 0\rangle}, x^{\langle 1\rangle}, \mathrm{fctn}\right)$$

The strength of each term is then deduced using a convenient function built into MathCAD...

$$\mathrm{dfctn}(\theta) := \begin{pmatrix} -7\sin(7\cdot\theta\cdot\mathrm{deg}) \\ -21\sin(3\cdot 7\cdot\theta\cdot\mathrm{deg}) \\ -35\sin(5\cdot 7\cdot\theta\cdot\mathrm{deg}) \\ -49\sin(7\cdot 7\cdot\theta\cdot\mathrm{deg}) \end{pmatrix}\cdot\frac{\mathrm{Wb}}{\mathrm{rad}}$$

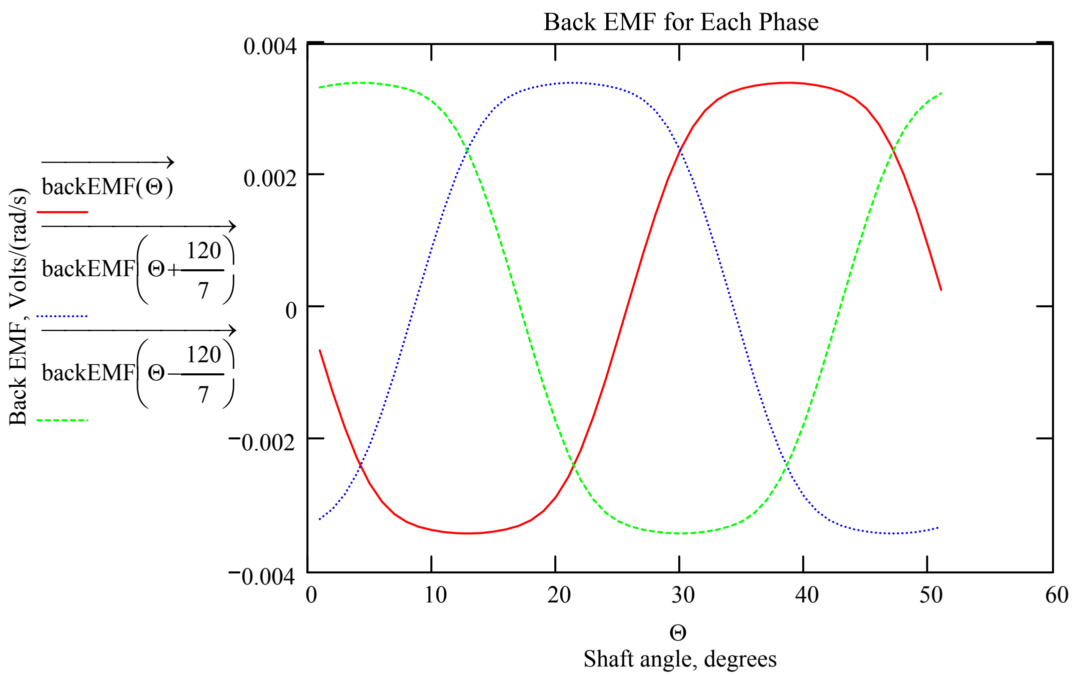
We can differentiate the terms in the Fourier series analytically and put them in dfctn.

$$\mathrm{backEMF}(\theta) := \mathrm{coeff}\cdot\mathrm{dfctn}(\theta)$$

And multiply the coefficients by the shape functions to get one analytical function for the back EMF.

$$\Theta := 1 .. \frac{360}{7}$$

Make a dummy array for the purposes of plotting back EMF over an entire wavelength



The K_p coefficient is the value of the back EMF at the top of the "plateau." Again, we can combine the regressed coefficients together in a simple way to get the value we are after.

$$K_p := \left|\mathrm{backEMF}\left(\frac{360}{28}\right)\right|$$

$$K_p = 3.413\times 10^{-3}\,\frac{\mathrm{N\cdot m}}{\mathrm{A}}$$

Cogging Torque

The magnets are distributed with their main component along the 7X harmonic. There are 12 poles. Since 12 and 7 don't have any common factors, we might expect that the cogging torque would lie predominantly along the 84X harmonic, repeating every 4.286 degrees.

The cogging torque was evaluated as part of the finite element analysis. With cogging torque, we are trying to compute a small torque with good precision, which is relatively difficult with finite elements. Since we have several cycles worth of cogging, we can get a good estimate of the magnitude of the cogging effect by fitting a sine with the expected period to the cogging force results

$$\text{cogfctn}(\theta) := \sin(12\cdot 7\cdot \theta\cdot \text{deg}) \qquad \text{cogcoeff} := \text{linfit}\left(x^{\langle 0\rangle}, x^{\langle 4\rangle}, \text{cogfctn}\right)\cdot \text{N}\cdot\text{m}$$

The amplitude of the cogging force is:

$$\text{cogcoeff} = 1.004\times 10^{-3}\,\text{N}\cdot\text{m}$$

We had previously estimated the maximum design force for the machine as:

$$\tau_{max} := 0.027465\cdot\text{N}\cdot\text{m}$$

So scaled to the maximum torque of the machine, the cogging is:

$$\text{cogcoeff} = 0.037\,\tau_{max}$$

We can plot the FEA and curve fit values to see if they are a reasonable match to one another.