LRK Motor Analysis Worksheet

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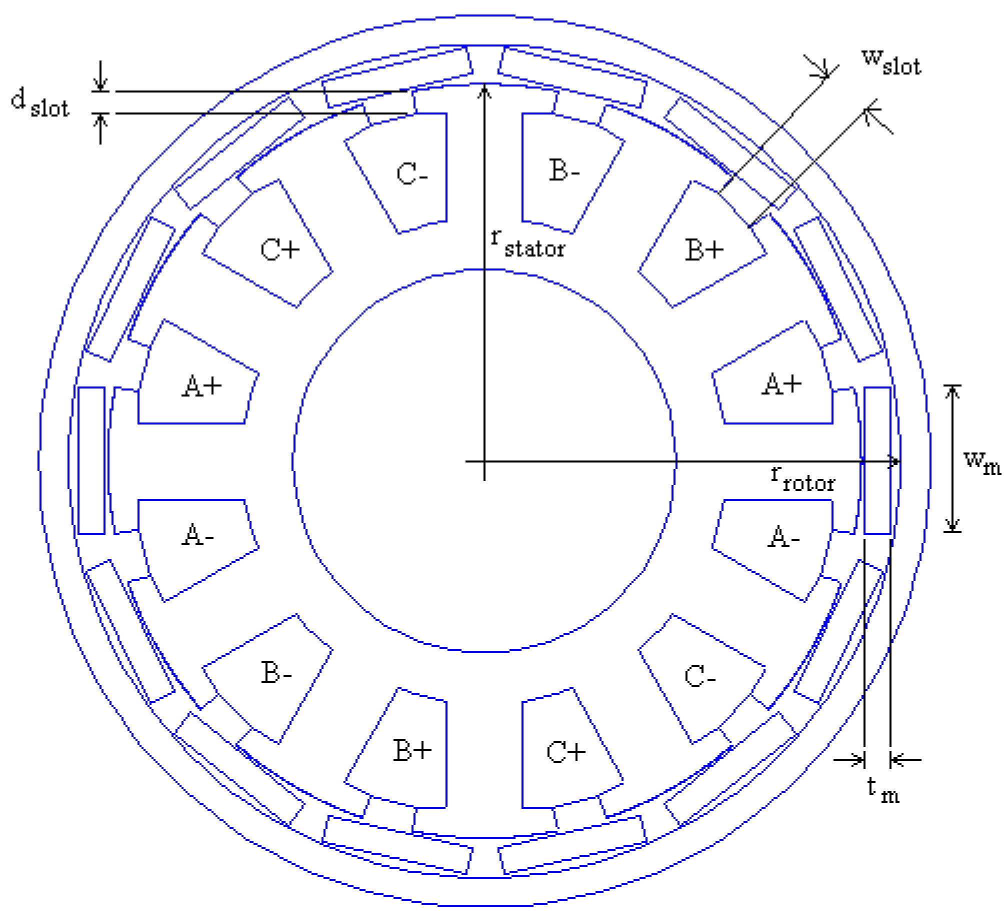
Introduction

This worksheet considers the analysis of a "LRK" motor. This type of motor is an "inside-out" machine with a rotor on the outside of the machine. The typical configuration, considered in this worksheet, has a 12-tooth stator and a 14 pole rotor. Only every other tooth of the stator is wound, and each winding only encircles a single tooth. Each phase contains a total of 2 coils, and these coils are wound in series. The topology, and some of the key dimensions, are pictured below.

The purpose of this worksheet is to predict the circuit parameters of the machine, ultimately allowing the voltage and current to be estmated for a desired torque and speed. The equations used in the worksheet are somewhat approximate, but they allow a design to be baselined before moving on to more elaborate analysis methods like FEA.



Preliminary Definitions:

$\mu_o := 4\cdot\pi\cdot 10^{-7}\cdot\frac{H}{m}$ $\qquad$ $MGOe := 10^6\cdot gauss\cdot oersted$ $\qquad$ $RPM := \frac{\pi\cdot rad}{30\cdot sec}$

$A_{rms} := \sqrt{2}\cdot A$ $\qquad$ $V_{rms} := \sqrt{2}\cdot V$

Machine Geometry:

See the above figure for each of the dimensions rendered on a drawing of the machine.

$r_{stator} := 12.8\cdot mm$	Outer radius of the stator
$r_{rotor} := 14.14\cdot mm$	Inner radius of the rotor
$t_m := 0.9\cdot mm$	Radial thickness of each magnet
$w_m := 5\cdot mm$	Width of each magnet
$n := 15$	Number of turns per coil
$p := 7$	Number of pole pairs on the rotor. Although the usual LRK machine has 7 pole pairs (i.e. 14 magnets on the rotor), the same stator configuration also supports a 5 pole pair rotor and generates more or less the same torque per amp*turn as the p=7 rotor.
$g_{tot} := \left(r_{rotor} - r_{stator}\right)$	Total distance between stator iron and rotor iron
$h := 5\cdot mm$	Axial length of the stator stack. It is usually a good idea to design the magnets so that they are of greater axial extent than the stator, to accommodate inevitable flux leakage in the axial direction from the permanent magnets.
$d_{slot} := 0.77\cdot mm$ $w_{slot} := 1.6\cdot mm$	These slot dimensions are used for computing leakage inductance. In this sort of machine, leakage accounts for a lot of the machine's inductance and needs to be considered to get a good estimate of the voltage drop over the internal impedance in the machine.

Magnet Properties:

$BH_{max} := 40 \cdot MGOe$ Energy product of the magnets

$B_r := \sqrt{4 \cdot \mu_o \cdot BH_{max}}$ Remanence of the permanent magnet, assuming that the magnet's permeability is the same as free space--an OK assumption for NdFeB or SmCo magnets.

$B_r = 1.264911\ T$

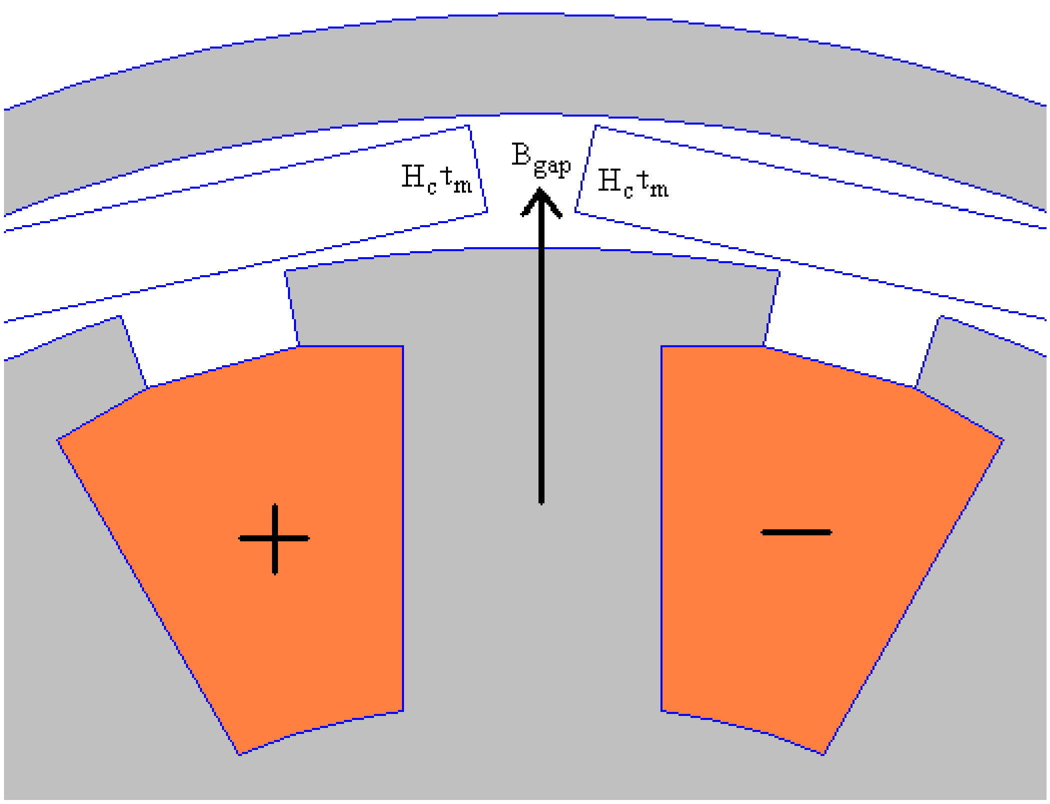
Wire Properties:

$\sigma := 58 \cdot 10^6 \cdot \frac{S}{m}$ Conductivity of the winding material. 58 MS/m is the conductivity of copper at room temperature.

$AWG := 22$ Gauge of wire used to wind the stator

Torque

To estimate torque, the easiest approach may be to consider the interaction of one tooth with the stator magnets when they are positioned above the tooth of interest so as to produce maximum torque. This configuration is shown in the picture below:



The trick is to remember that permanent magnets can be idealized as sheets of current that flow around the edges of an equivalent volume filled with air. We can use this idealization to compute torque on the magnets in a fairly straightforward way. The magnitude of the total current on each side of each magnet is Hc (the coercivity of the magnet) multiplied by tm, the thickness of the magnet. The force on a current subjected to a magnetic field (i.e. that produce by the coils in the stator) is i X B -- this is known as the "Lorentz force." Summing the contributions from each magnet, the i in the Lorentz force equation is 2*Hc*tm. We can get the field in the middle of the tooth due to the coil using magnetic circuit theory:

$$B = \mu_o * n * i / (r_{rotor} - r_{stator})$$

To get torque, we then need to multiply by a moment arm. which we could say is the center of the gap in which the magnets are located, $\left(r_{rotor} + r_{stator}\right)/2$. Multiplying all of this together, and multiplying bythe axial length of the machine, we get the single-pole result:

$$\tau_{onepole} = \left(\frac{r_{rotor} + r_{stator}}{r_{rotor} - r_{stator}}\right) \cdot B_r \cdot h \cdot t_m \cdot n \cdot i$$

To get the torque for the entire machine, we need to recall that there are 2 wound teeth per phase. Then, if we assume that the machine has a roughly trapezoidal back-EMF, so that we drive 2 out of the three phases full-on at any time, the total torque is 4X the one pole result.

$i_{phase} := 4.0 \cdot A$

Design current amplitude

$$K_p := 2 \cdot \left(\frac{r_{rotor} + r_{stator}}{r_{rotor} - r_{stator}}\right) \cdot B_r \cdot h \cdot t_m \cdot n$$

Where Kp represents the height of the plateau on a roughly trapezoidal back-EMF waveform for a phase.

$K_p = 3.433101 \times 10^{-3}\,Wb$

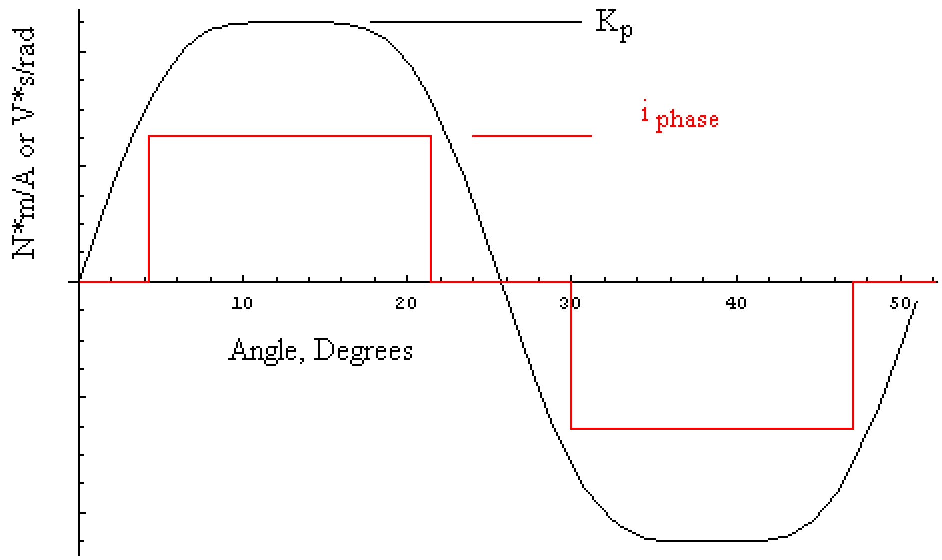
$\tau := 2K_p \cdot i_{phase}$

Torque produced by the motor when driven with 2 phases on and one off with the two "on" phases carrying currents of amplitude i_{phase}

$\tau = 0.027465\,N \cdot m$

$\tau = 0.243084\,lbf \cdot in$

An OK reference for terminology, etc., is:
http://www.eece.maine.edu/motor/BPMM_Ch8.pdf



Phase-to-Neutral Back-EMF waveform
and corresponding current waveform

Self-Inductance of Each Phase

$$gap := r_{rotor} - r_{stator}$$

$$R_{gap} := \frac{gap}{\mu_o \cdot \left[\frac{\pi \cdot (r_{rotor} + r_{stator})}{12}\right] \cdot (h + 2gap)}$$

Magnetic reluctance of the air gap between the rotor and stator. This uses an old but good kludge of augmenting the area of the flux path by the air gap width times the perimeter of the gap.

$$R_{leak} := \frac{w_{slot}}{\mu_o \cdot (d_{slot} + 2 \cdot w_{slot}) \cdot (h + 2 \cdot w_{slot})}$$

$$L_{gap} := \frac{2 \cdot n^2}{R_{gap}}$$

Inductance due to flux that crosses from the stator to the rotor and back

$$L_{gap} = 0.022858\,\mathrm{mH}$$

$$L_{leak} := \frac{4 \cdot n^2}{R_{leak}}$$

Leakage inductance from flux that crosses over to the neighboring unwound poles

$$L_{leak} = 0.023011\,\mathrm{mH}$$

$L_{phase} := L_{leak} + L_{gap}$

Inductance of each phase

$L_{phase} = 4.586942 \times 10^{-5}\,\mathrm{H}$

Resistance of Each Phase

$$d_{turn} := \frac{(2 \cdot \pi \cdot r_{stator})}{12}$$

End turn diameter. We will guesstimate that this is the same as the stator tooth pitch at the surface of the stator

$$l_{wire} := 2 \cdot n \cdot (\pi \cdot d_{turn} + 2 \cdot h)$$

Total length of wire in one phase, guesstimated to the circumference of a circle with the "end turn diameter" computed above, plus the length of wire required to run down one side of the slot and back the other. The leading 2 is because there are two n-turn coils per phase.

$$d_{wire}(awg) := (0.325105 \cdot in) \cdot e^{-0.115958 \cdot awg}$$

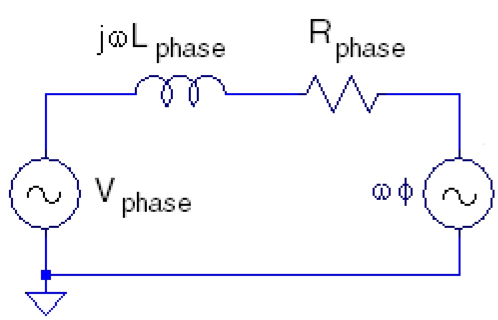
A convenient formula for wire diameter as a function of AWG wire gauge, regressed from a published table of wire sizes

$$a_{wire} := \frac{\pi\, d_{wire}(AWG)^2}{4}$$

$$R_{phase} := \frac{l_{wire}}{\sigma \cdot a_{wire}}$$

$R_{phase} = 0.049301\,\Omega$

Equivalent Circuit, Operating Point



This is a per-phase steady-state circuit model of the motor. It's useful for predicting voltage and power.

We could pretend that this is like a typical three-phase machine by computing the amplitude of the fundamentals of back-EMF and current, then plugging these in to get voltage, efficiency, etc.

$\phi := \left(\frac{2\cdot\sqrt{3}}{\pi}\right)\cdot\frac{K_p}{p}$ — Amplitude of the fundamental of the back-EMF of a trapezoidal waveform for one phase

$\omega_{mech} := 5000\cdot RPM$ — Mechanical speed at which the shaft is spinning

$\omega := p\cdot\omega_{mech}$ — Corresponding electrical frequency

$i_{fund} := \left(\frac{2\cdot\sqrt{3}}{\pi}\right)\cdot i_{phase}$ — Fundamental of the phase current waveform

We can then plug values into the circuit equation to get information about voltage, power, efficiency, etc.

$$V_{phase} := \left(j\cdot\omega L_{phase} + R_{phase}\right)\cdot i_{fund} + \omega\cdot\phi$$

$$V_{phase} = 2.199552 + 0.741516i\ V$$

$\left|V_{phase}\right| = 2.321179\ V$ — Phase Voltage Amplitude needed for the required torque

$V_{\Delta} := \left|\sqrt{3}\cdot V_{phase}\right|$ — Line-to-line voltage if the machine is Wye-configured. We could probably interpret this as the DC bus voltage for the two-phase-on system.

$$V_{\Delta} = 4.020401\ V$$

$P_{real} := \frac{3}{2}\cdot \mathrm{Re}\left(V_{phase}\cdot\overline{i_{fund}}\right)$ — Real power = mechanical power + losses

$$P_{real} = 14.552117\ W$$

$P_{apparent} := \frac{3}{2}\cdot\left|V_{phase}\right|\cdot\left|i_{fund}\right|$ — Apparent power = volt*amp product that must actually be accommodated by the drive electronics

$$P_{apparent} = 15.3568\ V\cdot A$$

$$\tau_{fund} := \frac{3}{2}\cdot p\cdot\phi\cdot i_{fund}$$

$P_{mech} := \tau_{fund}\cdot\omega_{mech}$ — Mechanical output power

$$P_{mech} = 13.113479\ W$$

$$\eta := \frac{P_{mech}}{P_{real}} \qquad \eta = 0.901139$$

Suggested Iron Cross-Section Sizing

$$B_{gap} := B_r \cdot \left(\frac{t_m}{r_{rotor} - r_{stator}} \right)$$

Air gap flux density

$$B_{max} := 1.8 \cdot T$$

$$w_{leg} := w_m \cdot \frac{B_{gap}}{B_{max}}$$

Minimum "Leg" region iron thickness required to carry the flux from the permanent magnet.

$$w_{leg} = 2.359909\,mm$$

$$w_{backiron} := \frac{w_{leg}}{2}$$

$$w_{backiron} = 1.179954\,mm$$

The back iron and rotor iron need about half the cross-section as the leg section to carry the flux from the magnets--in the case where all the flux goes down the "leg", half of the flux goes to the left, and half of the flux goes to the right....

Wire Sizing

$$i_{rms} := \sqrt{\frac{2}{3}} \cdot i_{phase}$$

Relation of RMS to peak current density for the two-phases-on strategy

$$\frac{i_{rms}}{a_{wire}} = 10.024119\,\frac{A}{mm^2}$$

Current density in the wire. As a rule of thumb, it's good to stay below 10 A/mm^2. However, the maximum current density is really determined by how well the design can get the heat produced via resistive losses out.

$$n \cdot a_{wire} = 4.887192\,mm^2$$

Cross-section area of copper in each slot. As another rule of thumb, it's good to make sure that the copper area is less than half of the total slot area so that the winding can actually be constructed.

Conclusions

The above represents a somewhat rough crack at analyzing an LRK motor designed to have a trapezoidal back-EMF and driven by a two-phase-on drive.

Several potentially important effects have been neglected. Efficiency should be expected to be worse than predicted by the worksheet, since core losses have been neglected. Eddy current losses in the magnets and solid rotor iron will also decrease efficiency to some degree, although this is probably a second-order effect compared to core losses in the stator iron.

Lastly, what would probably be even more useful is to put the math used in this worksheet together in a different order, so as to perform a design optimization. Probably the way to do this would be to prescribe:

- Maximum allowable average current density in the slots;
- Magnet grade;
- Operating speed;
- Torque;
- Drive voltage;
- Number of magnets (i.e. 10-pole or 14-pole design);
- Maximum allowable flux density;
- Geometric constraints (e.g. stator ID, axial length, etc.)

The program would then perform a minimization to obtain an optimal design, probably in the sense of minimizing weight, also selecting a wire gauge and turns count.