David Meeker
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Dcm3c@aol.com

Braking force in a Single-Sided Halbach Array Brake

Introduction

Consider the case where we have a stationary Halbach array. Each magnet in the array has a square l by l cross-section. Assume that each magnet has a coercivity of H_c and a unit relative permeability. A plate moves below the array at a constant speed of v. The plate's thickness is $2w$, and the distance from the surface of the Halbach array to the center of the plate is denoted as g. The plate also has a unit relative permeability in addition to a conductivity of σ.

Field inside the plate

To get simple expressions, one can assume that the plate is actually a current sheet. This yields pretty good results as long as the plate is thin relative to the pole pitch. Assume that this current sheet lies along the line $y = 0$.

The differential form of Ampere's loop law is $\nabla \times H = J$. If we integrate this expression across the plate's thickness, the result is

$$-\left(H_x[0+] - H_x[0-]\right) = K$$

where K is the current in the current sheet. The definition of vector potential is $B = \nabla \times A$. In combination with the constitutive law $B = \mu_o H$, we could write the cross-plate condition as:

$$\frac{\partial A}{\partial y}[0+] - \frac{\partial A}{\partial y}[0-] = -\mu_o K$$

This condition on the derivatives, in combination with the continuity of A, must be obeyed as the field crosses the plate.

For our purposes, it is useful to consider just the fundamental of the field produced by the magnets. In this case, we can represent vector potential A as:

$$A = \mathrm{Re}\left[a\left(\cos[\beta x] + j\sin[\beta x]\right)\right]$$

where β is the wavenumber of the Halbach array:

$$\beta = \frac{\pi}{2l}$$

We could then split the field into two parts:

$$a = a_s + a_r$$

where a_s is the field due to the Halbach array in the absence of the plate, and a_r is the reaction field from the currents in the plate.

The current in the plate is due to the motion of the plate in the field of the Halbach array. The current density is:

$$J = \sigma v \times B$$

If we integrate the current across the plate and re-write in terms of a, we get:

$$k = -(2 j\beta v\sigma w)(a_s + a_o)$$

Outside the plate, the equation that the magnetic field must then obey is:

$$\frac{d^2 a}{dy^2} - \beta^2 a = 0$$

which can be solve by inspection to yield a

$$a_r = a_o e^{-\beta y} \text{ for } y > 0$$
$$a_r = a_o e^{\beta y} \text{ for } y < 0$$

The interface condition can then be used to determine the unknown constant, a_o:

$$-\beta a_o - \beta a_o = (2 j\beta v\sigma\mu_o w)(a_s[0] + a_o)$$

For ease of notation, we can define the "modified slip" as:

$$s_m = v\sigma\mu_o w$$

and we can also note that the time constant of the plate then must be:

$$\tau = \frac{\sigma\mu_o w}{\beta}$$

Now, we can solve for a_o as a function of a_s:

$$a_o = \left(\frac{-js_m}{1+js_m}\right)a_s[0]$$

and the total *a* inside the plate as:

$$a = \left(\frac{1}{1+js_m}\right)a_s[0]$$

The above represents the field as a simple function of the plate's velocity and the "source" field produced by the halbach array magnets.

Forces on the plate

The best way to get force in this case is to integrate the $J \times B$ force across the plate to get the pressure:

$$P = -\frac{1}{2}\mathrm{Re}\left[b_y \bar{k}\right]$$

The flux passing normal to the plate is:

$$b_y = -j\beta a = \frac{-j\beta}{1+js_m}a_s$$

and the current flowing in the plate is:

$$k = -\frac{2j\beta s_m a}{\mu_o} = -\frac{1}{\mu_o}\frac{2j\beta s_m}{1+js_m}a_s$$

Using these expressions to evaluate the pressure yields:

$$P = -\left(\frac{s_m}{1+s_m{}^2}\right)\frac{(\beta a_s)^2}{\mu_o}$$

The total force would then be obtained by multiplying by the surface area of the Halbach array.

Right away, this form give some interesting results. By inspection, one can see that the braking pressure is optimized at $s_m = 1$. At the peak of the curve, the velocity is

$$v = \frac{1}{w\sigma\mu_o}$$

and the peak pressure is:

$$P = \frac{(\beta a_s)^2}{2\mu_o}$$

Field from the Halbach array

The last part that remains to be determined is a_s, the field component at the plate's center due to the Halbach array in free space. If one considers just the fundamental of the array, the array can be represented as a sinusoidally distributed current density of amplitude J_h sandwiched between two current sheets of strength K_h on the bottom of the array and $-K_h$ on the top of the array. Creating J_h and K_h just consists of evaluating Fourier series coefficients to obtain:

$$J_h = \frac{2\sqrt{2}}{\pi}\beta H_c$$

$$K_h = \frac{2\sqrt{2}}{\pi}H_c$$

One then needs to solve:

$$\frac{d^2 a_s}{dy^2} - \beta^2 a_s = -\mu_o\left(K_h\left(\delta[y-g] - \delta[y-g-l]\right) + J_h\left(u[y-g] - u[y-g-l]\right)\right)$$

where $u[y]$ and $\delta[y]$ are the unit step and Dirac delta functions respectively. The implication here is that the array is located a distance of g above the plate's center.

Skipping over the solution of this ODE, the field at the center of the plate is:

$$a_s = \left(\frac{4\sqrt{2}\sinh\left[\frac{\pi}{4}\right]\exp\left[-\left(\beta g + \pi/4\right)\right]\mu_o H_c}{\beta\pi}\right)$$

Example

As a giggle test, one can compare to a finite element run. In this case, the magnets have $H_c = 10^6$ A/m, $l = 0.75$", g=0.1875", w=0.0625", σ=25 MS/m. The plate is traveling past at 4.572 m/s, which corresponds to a frequency of 120 Hz and a s_m of 0.456. A section 1.5" long in the travel direction has been modeled.

The finite element solution yields an average force of 4327 N/m; the "simple" analytical formula predicts a force of 4191 N/m.

Conclusions

By assuming that the plate can be approximated as current sheet, some fairly simple expressions for braking pressure versus speed can be derived for a single-sided brake.   In these formulas, the magnet geometry is clearly in evidence, and simple expressions are available for the peak braking force and the velocity at which it occurs. The answer shows a good agreement to a finite element analysis for a rather arbitrary test case (~3% difference between analytical solution and FEA).

This solution does not include skin effects. Plates that are thick compared to the wavelength are also not addressed. It would be straightforward to extend the solution to encompass these cases as well, with the expense of some loss of simplicity. A double-sided brake could be accommodated more easily (just scales force by a factor of 4).

It should also be noted that the above is a purely 2D solution. In the "real world," the eddy currents have to "turn around" in at the edges of the plate—this creates some extra resistance and inductance. Most of the effect can be captured by a relatively simple correction to the plate's conductivity that accounts for the increased resistance.