Since all the materials in your problem are linear, there must be a linear relationship between the voltage and currents. We could write out this relationship as:

$$\begin{bmatrix} Z_1 & Z_2 \\ Z_2 & Z_3 \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} = \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

In this relationship, the Z matrix represents the self and coupled impedances of the transformer; i_1 and i_2 represent the currents through the top and bottom windings, respectively, and v_1 and v_2 represent the voltages across the top and bottom windings, respectively.

Now, FEMM can only model situations in which the currents are known. The voltages that drive these currents can then be inferred. In the case of a transformer, the situation is reversed: we know the voltage drop across primary, and in this particular case of a shorted secondary, we know the voltage across the load attached to the secondary (*i.e.* zero).

Since we know voltages but not currents, we can't *a priori* make a single model that models the short-circuit operation of the transformer. Instead, we make a series of models to identify the impedances Z_1, Z_2, and Z_3. We can then say what currents are associated with a particular set of applied voltage by inverting the impedance matrix:

$$\begin{Bmatrix} i_1 \\ i_2 \end{Bmatrix} = \left(\frac{1}{Z_1 Z_3 - Z_2^{\,2}} \right) \begin{bmatrix} Z_3 & -Z_2 \\ -Z_2 & Z_1 \end{bmatrix} \begin{Bmatrix} v_1 \\ v_2 \end{Bmatrix}$$

These currents could then be applied to the model to get a model of the transformer under short-circuit conditions. However, once the impedance matrix is obtained, there may be no need for further analyses—losses and couplings can be inferred from the impedance matrix.

To get the impedance parameters, first make a model with a circuit for each turn in the primary, where each turn carries 1 Amp. For the each turn of the secondary, make a circuit such that each turn carries 0 Amps. This configuration is reflected in the attached Case1.fem. Analyze the model and run the post-processor. To get the Z_1 term, one simply sums up the impedances that are reported for each turn of winding 1 that are reported the View|Circuit Props. One could also automate this via the short Lua script:

```
ztim=0
ztre=0
for i=1,7,1 do
v1,v2,v3,v4,zre,zim=getcircuitproperties("c"..i)
ztre=ztre+zre
ztim=ztim+zim
end
print(ztre,ztim)
```

which can be pasted into the Lua console window. The script prints the summed impedances. The result of the run is:

$$Z_1 = 0\ 0.02716250196921174 + j1.252897988452293\ \Omega$$

Looking at the matrix representation, one can see that the voltage across the turns of the second coil can be summed to yield the Z_2 cross-coupling term. A script that computes this coupling is:

```
ztim=0
ztre=0
for i=1,4,1 do
    v1,v2,v3,v4=getcircuitproperties("CC"..i)
    ztre=ztre+v3
    ztim=ztim+v4
end
print(ztre,ztim)
```

The result of this calculation is:

$$V = -0.0001958265023914235 - j0.07370789290762653 \text{ Volts/rad}$$

Note that this result is in volts per radian, so we need to multiply by 2*Pi to get back to volts. The sign convention used internally is such that the Z_2 impedance is:

$$Z_2 = 0.00123041 + j\ 0.46312\ \Omega$$

A second simulation can then be run with all of the turns in the second coil carrying 1 Amp and all of the turns in the first coil carrying 0 Amps. This is the attached Case2.fem.

```
ztim=0
ztre=0
for i=1,4,1 do
    v1,v2,v3,v4,zre,zim=getcircuitproperties("CC"..i)
    ztre=ztre+zre
    ztim=ztim+zim
end
print(ztre,ztim)
```

The result of the calculation is:

$$Z_3 = 0.01199329261144129 + j\ 0.4294433540007501$$

You could then calculate some stuff like coupling coefficients…. You could decompose the impedance matrix as:

$$Z = j\omega L + R$$

where L and R are real-valued matrices of apparent inductance and resistance. Note that R has nonzero cross-coupling terms due to the proximity effects. For this case, $\omega = 50000\pi$ rad/sec. The respective matrices are:

$$L = \begin{bmatrix} 7.9762 & 2.9483 \\ 2.9483 & 2.7339 \end{bmatrix} \ \mu\text{H}$$

$$R = \begin{bmatrix} 27.1625 & 2.9483 \\ 2.9483 & 11.9933 \end{bmatrix} \ \text{m}\Omega$$

For example, the coupling is:

$$k = \sqrt{\frac{L_{1,2} L_{2,1}}{L_{1,1} L_{2,2}}} = 0.631669$$

Note that these results apply only at the one frequency at which the problem was evaluated. The apparent inductance and resistance will vary with frequency.