Determination of Induction Motor Operating Point

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From the finite element analysis, we have identified the following motor parameters:

$L_l := 16.3 \cdot mH$ Stator leakage inductance

$M := 316.4 \cdot mH$ Mutual inductance between the rotor and stator

$\tau_{fea} := 0.163 \cdot s$ Rotor time constant determined from FEA with room-temperature material properties.

$R_r := \dfrac{M}{\tau_{fea}}$ Room-temperature rotor resistance

$R_r = 1.941\,\Omega$

Assume a 60C temperature rise:

$R_r := R_r \cdot \left(1 + 60 \cdot \dfrac{0.4}{100}\right)$

$MS := 10^6 \cdot S$

$R_r = 2.407\,\Omega$

Assume that our geometry is such that the end bars of the rotor tack on another 50% to the rotor resistance (could actually calculate this carefully if we knew the end bar geometry, but this is just meant to be an illustrative example...)

$R_r := 1.50 R_r$ Increase in resistance due to end bars

This is 4 pole machine, so the number of pole pairs is: $p := 2$

We define the rotor time constant to be the quantity: $\tau := \dfrac{M}{R_r}$

The rotor time constant of the "real" machine is a lot shorter than we'd expect from the 2D finite element analysis:

$$\frac{\tau}{\tau_{fea}} = 0.538$$

One crucial parameter that we do not have yet is a value for rotor resistance. However, we can get a good estimate of this value.

$\sigma := 48 \cdot 10^{6} \cdot \frac{S}{m}$ The value that we assume for the resistivity of copper. This resistance corresponds to copper at roughly 80 C

$a_{slot} := 57.46 \cdot mm^{2}$ This is the area of the stator slot openings (can get from FEA geometry)

$n_{slot} := 44$ The number of turns per slot (we assumed this)

What gauge wire is the stator wound with? The cross-section area of an 20 AWG wire strand is:

$$a_{wire} := \frac{\pi}{4}(0.032 \cdot in)^{2}$$

With the 20 AWG wire, the implied slot fill factor is

$$\frac{n_{slot} \cdot a_{wire}}{a_{slot}} = 0.397$$

$h := 100 \cdot mm$ Length of the machine in the into-the-page direction

Now it is easy enough to account for the resistance contributions of the copper in the slots. However, a big portion of the stator resistance is due to the end-turn wire that runs between the slots. We have to guesstimate some contribution of the end-turns.

Looking at the geometry, sort of an "average radius" of the windings would be about 50 mm:

$r_{avg} := 50 \cdot mm$

We might then assume that for turn of wire each slot, there is an additional length of wire equal to 3/2 r_{avg} that connects the wire to the turn in the next slot in a roughly half-circular arc. This isn't really a hard-and-fast design formula, but just sort of a guess for our present purposes.

$slots\_per\_phase := 12$ Number of slots attributed to each of the 3 phases

The total length of wire used to wind any one phase is then:

$$\text{wire\_length} := n_{slot} \cdot \text{slots\_per\_phase} \cdot \left(h + \frac{3}{2} \cdot r_{avg} \right)$$

$$R_s := \frac{\text{wire\_length}}{a_{wire} \cdot \sigma}$$

We could now write a formula for the resistance of one stator phase

$$R_s = 3.71\,\Omega$$

We are going to operate our motor off of a 220 Vrms, 3-phase, 50 Hz supply. Since our motor is delta connected:

$$v_{phase} := 220 \cdot V$$

$$\omega := 50 \cdot Hza$$

As we had derived earlier, the total impedance of the motor as seen from the terminals is:

$$Z(\omega_s) := R_s + j \cdot \omega \cdot L_l + j \cdot \omega \cdot \frac{M}{1 + j \cdot \tau \cdot \omega_s}$$

where we have written this impedance as a function of slip frequency ω_s

The current that is then generated at a given slip frequency is:

$$i_{phase}(\omega_s) := \frac{v_{phase}}{Z(\omega_s)}$$

Using our formula for torque as a function of slip and phase current,

$$Tq(\omega_s) := 3 \cdot p \cdot M \cdot \left(\left| i_{phase}(\omega_s) \right| \right)^2 \cdot \frac{\tau \cdot \omega_s}{1 + \left(\tau \cdot \omega_s \right)^2}$$

Output power is then the product of torque and mechanical speed:

$$\omega_{mech}(\omega_s) := \frac{\omega - \omega_s}{p}$$

$$P(\omega_s) := Tq(\omega_s)\cdot\omega_{mech}(\omega_s)$$

We then solve for the slip frequency at which the output power is equal to the output power that we require:

$$\omega_s := 2.356\cdot Hza$$

$$P(\omega_s) = 2\ \text{hp}$$

Now that we know the operating point, we can compute some details about it:

$$rpm := 2\cdot\pi\cdot\frac{rad}{min}$$

The mechanical speed at load is:

$$\omega_{mech}(\omega_s) = 1429.32\ \text{rpm}$$

The line current is:

$$i_{line} := \sqrt{3}\cdot\left|i_{phase}(\omega_s)\right| \qquad \tau\cdot\omega_s = 1.297$$

$$i_{line} = 5.707\ \text{A}$$

The power factor of the machine is:

$$i_p := i_{phase}(\omega_s)$$

$$X := 3\ i_{phase}(\omega_s)\cdot v_{phase}$$

$$\frac{Re(X)}{|X|} = 0.775$$

Efficiency of the machine is:

$$\frac{P(\omega_s)}{Re(X)} = 0.885$$

This seems abnormally high, but we neglected iron losses, windage and friction losses, and "stray load loss." These effects will drop the efficiency somewhat. We'd expect an efficiency in the neighborhood of 77% to 82% for a 2 HP motor, depending on the design.

We could go back and add a resistor in parallel to the mutual inductance to model the eddy current losses in our circuit. We could model hysteresis losses by making the mutual inductance complex-valued. We could make some model of the mechanical friction and windage, ultimately requiring more output than 2 hp to overcome this mechanical drag. However, this is all beyond the scope of this simple example...

Where is the loss that we have accounted for coming from?

$$i_r := \left| \frac{j \cdot \omega_s \cdot \tau}{1 + j \cdot \omega_s \cdot \tau} \right| \cdot i_p$$ Rotor current magnitude

$$3 \cdot \left(\left| i_r \right| \right)^2 R_r = 73.751\ \mathrm{W}$$ Rotor resistive losses

$$3 \cdot \left(\left| i_p \right| \right)^2 \cdot R_s = 120.816\ \mathrm{W}$$ Stator winding losses

Lastly, it would be nice to do a simulation at the operating point, if only for getting some sort of a handle on iron losses. We can't actually analyze motion with FEMM, but from our circuit model, we can see that we could get similar results if we scale the resistivity of the rotor bars so that we have an effective bar resistance that is equal to ω*R/ωs

$$\sigma_{effective} := \frac{34.45 \cdot \frac{10^6 S}{m}}{\left[\frac{\omega}{\omega_s} \cdot 1.25 \cdot \left(1 + 60 \cdot \frac{0.4}{100} \right) \right]}$$

This is the effective resistance that we will apply to the rotor bars to do the on-load simulation.

$$\sigma_{effective} = 1.047 \frac{\mathrm{MS}}{\mathrm{m}}$$

$$\frac{\omega_s}{\omega} = 0.047$$

We would also need to scale the conductivity of the rotor lamiations by this factor, since the rotor lams just see the slip frequency, rather than the fundamental.

Note that any loss results will be approximate and not conservative. Losses associated with harmonics that the machine sees when the rotor is spinning aren't captured by this model and will tend to increase losses.