Refer to https : // www.femm.info/wiki/AxisymmetricFormulation for nomenclature. Copied here for convenience...

FEMM Axisymmetric Interpolation Functions and Flux Density

David Meeker
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Introduction

The purpose of this note is to explain the somewhat unusual formulation used in FEMM's axisymmetric formulation. This is basically the method for first-order triangles considered in Section IV of Henrotte *et al.* The basic difference between this and other first-order methods is that this method interpolates radially in terms of r^2, rather than in terms of just r. The result is a formulation that is more physically consistent than many other competing methods, yielding an accurate solution close to $r = 0$ where other methods can break down.

Interpolation

Instead of the regular magnetic vector potential A, the nodal field values are in terms of nodal flux ϕ, where A and ϕ are related by:

$$\phi = 2\pi r A$$

For ease of notation, define:

$$s = r^2$$

Now, consider a "unit triangle" that lies between the points $(p, q) = (0, 0), (1, 0), (0, 1)$ as pictured in Figure 1.

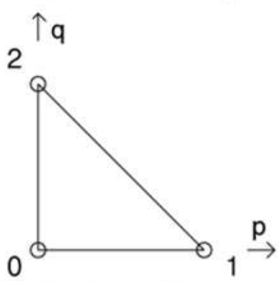

Figure 1: Unit triangle.

We can map the (p, q) coordinates back to the physical (s, z) coordinates usina standard linear interpolation scheme:

$$s = (s_1 - s_0)p + (s_2 - s_0)q + s_0$$
$$z = (z_1 - z_0)p + (z_2 - z_0)q + z_0$$

Nodal ϕ is interpolated in a similar way as:

$$\phi = (\phi_1 - \phi_0)p + (\phi_2 - \phi_0)q + \phi_0$$

This relationship between element and physical coordinates can be inverted to yield:

$$\begin{Bmatrix} p \\ q \end{Bmatrix} = \frac{1}{d} \begin{bmatrix} (z_2 - z_0) & -(s_2 - s_0) \\ -(z_1 - z_0) & (s_1 - s_0) \end{bmatrix} \begin{Bmatrix} s - s_0 \\ z - z_0 \end{Bmatrix}$$

where

$$d = (s_1 - s_0)(z_2 - z_0) - (z_1 - z_0)(s_2 - s_0)$$

We could write this more compactly as:

$$\begin{Bmatrix} p \\ q \end{Bmatrix} = \frac{1}{d} \begin{bmatrix} b_1 & c_1 \\ b_2 & c_2 \end{bmatrix} \begin{Bmatrix} s - s_0 \\ z - z_0 \end{Bmatrix}$$

where

$$d = c_2 b_1 - b_2 c_1$$

where the $b's$ and $c's$ are the corresponding entries in the previous matrices.

It is interesting to note that interpolating s rather than r produces an element with "warped" sides in the rz plane.

Flux Density

In terms of ϕ, the flux density can be written as:

$$B_r = -\frac{1}{2\pi r}\frac{\partial \phi}{\partial z} \quad B_z = \frac{1}{2\pi r}\frac{\partial \phi}{\partial r}$$

To be easily integrated with the fashion in which we are interpolating the element, the derivatives can be re-written as in terms of s rather than r as:

$$B_r = -\frac{1}{2\pi\sqrt{s}}\frac{\partial \phi}{\partial z} \quad B_z = \frac{1}{\pi}\frac{\partial \phi}{\partial s}$$

Writing out the derivatives of the interpolation function yields:

$$\left\{ \begin{matrix} \frac{\partial \phi}{\partial z} \\ \frac{\partial \phi}{\partial z} \end{matrix} \right\} = \left\{ \begin{matrix} \frac{\partial \phi}{\partial q}\frac{\partial q}{\partial z} + \frac{\partial \phi}{\partial p}\frac{\partial r}{\partial z} \\ \frac{\partial \phi}{\partial q}\frac{\partial q}{\partial z} + \frac{\partial \phi}{\partial p}\frac{\partial r}{\partial z} \end{matrix} \right\} = \frac{1}{d}\begin{bmatrix} b_1 & b_2 \\ c_1 & c_2 \end{bmatrix} \left\{ \begin{matrix} \phi_1 - \phi_0 \\ \phi_2 - \phi_0 \end{matrix} \right\}$$

Substituting into the formulas for the derivatives yields:

$$B_r = -\frac{1}{2\pi d\sqrt{s}}\left(c_1(\phi_1 - \phi_0) + c_2(\phi_1 - \phi_0)\right)$$

$$B_r z = -\frac{1}{\pi d}\left(b_1(\phi_1 - \phi_0) + b_2(\phi_1 - \phi_0)\right)$$

Interpretation of the form of B

Now, we can observe that B_z is constant over the element, which is a reasonable behavior for a first-order element. The B_r flux becomes smaller as r increases, but the $1/r$ scaling factor is consistent with a constant amount of radially directed flux being spread over a larger area as the radius increases. This form is similar to the standard 3-node triangle formulation for 2-D planar problems, in which flux is constant over each element.

References

F. Henrotte *et al.*, "A new method for axisymmetric linear and nonlinear problems," *IEEE Transactions on Magnetics*, 9(2):1352-1355, March 1993. doi: 10.1109/CEFC.1992.720582

In[1]:= **Br = -1 / (2 Pi r d) (c1 (ϕ1 - ϕ0) + c2 (ϕ2 - ϕ0))**

Out[1]= $-\frac{c1\,(-\phi0 + \phi1) + c2\,(-\phi0 + \phi2)}{2\,d\,\pi\,r}$

In[2]:= **Bz = 1 / (Pi d) (b1 (ϕ1 - ϕ0) + b2 (ϕ2 - ϕ0))**

Out[2]= $\frac{b1\,(-\phi0 + \phi1) + b2\,(-\phi0 + \phi2)}{d\,\pi}$

In[3]:= **r = Sqrt[(s1 - s0) p + (s2 - s0) q + s0];**

In[4]:= **dW = (1 / 2) * (Br^2 / μr + Bz^2 / μz)**

Out[4]= $\frac{1}{2}\left(\frac{(b1\,(-\phi0 + \phi1) + b2\,(-\phi0 + \phi2))^2}{d^2\,\pi^2\,\mu z} + \frac{(c1\,(-\phi0 + \phi1) + c2\,(-\phi0 + \phi2))^2}{4\,d^2\,\pi^2\,(s0 + p\,(-s0 + s1) + q\,(-s0 + s2))\,\mu r}\right)$

We want to integrate 2 Pi r dW dr dz. Note that dr = ds*(dr/ds) = ds*D[Sqrt[s],s] = ds/(2*Sqrt[s]) =ds/(2*r) so that 2 Pi r dW dr dz = Pi dW ds dz. To integrate over p and q, turns into Pi dW d dp dq

In[5]:= **X = FullSimplify[Integrate[FullSimplify[Pi dW d], {p, 0, 1 - q}]][[1]]**

Out[5]= $\frac{-\frac{4\,(-1+q)\,(b1\,(\phi0-\phi1)+b2\,(\phi0-\phi2))^2}{\mu z} + \frac{(c1\,(\phi0-\phi1)+c2\,(\phi0-\phi2))^2\,(\text{Log}[s0-q\,s0+q\,s2]-\text{Log}[s1-q\,s1+q\,s2])}{(s0-s1)\,\mu r}}{8\,d\,\pi}$

In[6]:=
```
Y = Integrate[X, {q, 0, 1}][[1]]
```

Out[6]=
$$\frac{\frac{2\,(b1\,(\phi0-\phi1)+b2\,(\phi0-\phi2))^2}{\mu z}+\frac{(c1\,(\phi0-\phi1)+c2\,(\phi0-\phi2))^2\,(s0\,(-s1+s2)\,\mathrm{Log}[s0]+s1\,(s0-s2)\,\mathrm{Log}[s1]+(-s0+s1)\,s2\,\mathrm{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi}$$

Y is an expression for the stored energy in an element. Differentiate with respect to the nodal values of flux to get the element matrices

In[7]:=
```
m = {D[Y, ϕ0], D[Y, ϕ1], D[Y, ϕ2]}
```

Out[7]=
$$\Bigg\{\frac{1}{8\,d\,\pi}\Bigg(\frac{4\,(b1+b2)\,(b1\,(\phi0-\phi1)+b2\,(\phi0-\phi2))}{\mu z}+ (2\,(c1+c2)\,(c1\,(\phi0-\phi1)+c2\,(\phi0-\phi2))\,(s0\,(-s1+s2)\,\mathrm{Log}[s0]+s1\,(s0-s2)\,\mathrm{Log}[s1]+ (-s0+s1)\,s2\,\mathrm{Log}[s2]))\,/\,((s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r)\Bigg),$$
$$\frac{-\frac{4\,b1\,(b1\,(\phi0-\phi1)+b2\,(\phi0-\phi2))}{\mu z}-\frac{2\,c1\,(c1\,(\phi0-\phi1)+c2\,(\phi0-\phi2))\,(s0\,(-s1+s2)\,\mathrm{Log}[s0]+s1\,(s0-s2)\,\mathrm{Log}[s1]+(-s0+s1)\,s2\,\mathrm{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi},$$
$$\frac{-\frac{4\,b2\,(b1\,(\phi0-\phi1)+b2\,(\phi0-\phi2))}{\mu z}-\frac{2\,c2\,(c1\,(\phi0-\phi1)+c2\,(\phi0-\phi2))\,(s0\,(-s1+s2)\,\mathrm{Log}[s0]+s1\,(s0-s2)\,\mathrm{Log}[s1]+(-s0+s1)\,s2\,\mathrm{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi}\Bigg\}$$

In[27]:=
```
M = {D[m, ϕ0], D[m, ϕ1], D[m, ϕ2]}
```

Out[27]=

$$\left\{\left\{\frac{\frac{4\,(b1+b2)^2}{\mu z}+\frac{2\,(c1+c2)^2\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi},\right.\right.$$

$$\frac{-\frac{4\,b1\,(b1+b2)}{\mu z}-\frac{2\,c1\,(c1+c2)\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi},$$

$$\left.\frac{-\frac{4\,b2\,(b1+b2)}{\mu z}-\frac{2\,c2\,(c1+c2)\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi}\right\},$$

$$\left\{\frac{-\frac{4\,b1\,(b1+b2)}{\mu z}-\frac{2\,c1\,(c1+c2)\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi},\right.$$

$$\frac{\frac{4\,b1^2}{\mu z}+\frac{2\,c1^2\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi},$$

$$\left.\frac{\frac{4\,b1\,b2}{\mu z}+\frac{2\,c1\,c2\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi}\right\},$$

$$\left\{\frac{-\frac{4\,b2\,(b1+b2)}{\mu z}-\frac{2\,c2\,(c1+c2)\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi},\right.$$

$$\frac{\frac{4\,b1\,b2}{\mu z}+\frac{2\,c1\,c2\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi},$$

$$\left.\left.\frac{\frac{4\,b2^2}{\mu z}+\frac{2\,c2^2\,(s0\,(-s1+s2)\,\text{Log}[s0]+s1\,(s0-s2)\,\text{Log}[s1]+(-s0+s1)\,s2\,\text{Log}[s2])}{(s0-s1)\,(s0-s2)\,(-s1+s2)\,\mu r}}{8\,d\,\pi}\right\}\right\}$$

Split into element matrices for the r - and z - directions

In[9]:=
```
Mz = FullSimplify[Limit[M , μr → Infinity]] /. b1 + b2 → -b0;
MatrixForm[Mz]
```

Out[10]//MatrixForm=

$$\begin{pmatrix} \frac{b0^2}{2\,d\,\pi\,\mu z} & \frac{b0\,b1}{2\,d\,\pi\,\mu z} & \frac{b0\,b2}{2\,d\,\pi\,\mu z} \\ \frac{b0\,b1}{2\,d\,\pi\,\mu z} & \frac{b1^2}{2\,d\,\pi\,\mu z} & \frac{b1\,b2}{2\,d\,\pi\,\mu z} \\ \frac{b0\,b2}{2\,d\,\pi\,\mu z} & \frac{b1\,b2}{2\,d\,\pi\,\mu z} & \frac{b2^2}{2\,d\,\pi\,\mu z} \end{pmatrix}$$

In[24]:=
```
Mr = FullSimplify[Limit[M , μz → Infinity]] /. c1 + c2 → -c0
```

Out[24]=

$$\left\{\left\{\frac{c0^2\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c0\,c1\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c0\,c2\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r}\right\},\left\{\frac{c0\,c1\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c1^2\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c1\,c2\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r}\right\},\left\{\frac{c0\,c2\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c1\,c2\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c2^2\,(s0\,(s1-s2)\,\mathrm{Log}[s0]+s1\,(-s0+s2)\,\mathrm{Log}[s1]+(s0-s1)\,s2\,\mathrm{Log}[s2])}{4\,d\,\pi\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r}\right\}\right\}$$

However, I want to implement the code in terms of vector potential, A, for various reasons. We can convert the element matrices for use with nodal values of A with the matrix T, i.e. {ϕ0,ϕ1,ϕ2}=T.{A0,A1,A3} where T is:

In[13]:=
```
T = 2 Pi DiagonalMatrix[{r0, r1, r2}]
```

Out[13]=
```
{{2 π r0, 0, 0}, {0, 2 π r1, 0}, {0, 0, 2 π r2}}
```

Z - direction element matrix for use with vector potential is :

In[14]:=
```
Mza = T.Mz.T;
MatrixForm[Mza]
```

Out[15]//MatrixForm=

$$\begin{pmatrix} \frac{2\,b0^2\,\pi\,r0^2}{d\,\mu z} & \frac{2\,b0\,b1\,\pi\,r0\,r1}{d\,\mu z} & \frac{2\,b0\,b2\,\pi\,r0\,r2}{d\,\mu z} \\ \frac{2\,b0\,b1\,\pi\,r0\,r1}{d\,\mu z} & \frac{2\,b1^2\,\pi\,r1^2}{d\,\mu z} & \frac{2\,b1\,b2\,\pi\,r1\,r2}{d\,\mu z} \\ \frac{2\,b0\,b2\,\pi\,r0\,r2}{d\,\mu z} & \frac{2\,b1\,b2\,\pi\,r1\,r2}{d\,\mu z} & \frac{2\,b2^2\,\pi\,r2^2}{d\,\mu z} \end{pmatrix}$$

R - direction element matrix for use with vector potential is :

In[25]:= **Mra = T.Mr.T**

Out[25]=

$$\left\{\left\{\frac{c0^2\,\pi\,r0^2\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c0\,c1\,\pi\,r0\,r1\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c0\,c2\,\pi\,r0\,r2\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r}\right\},\left\{\frac{c0\,c1\,\pi\,r0\,r1\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c1^2\,\pi\,r1^2\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c1\,c2\,\pi\,r1\,r2\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r}\right\},\left\{\frac{c0\,c2\,\pi\,r0\,r2\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c1\,c2\,\pi\,r1\,r2\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r},\frac{c2^2\,\pi\,r2^2\,(s0\,(s1-s2)\,\text{Log}[s0]+s1\,(-s0+s2)\,\text{Log}[s1]+(s0-s1)\,s2\,\text{Log}[s2])}{d\,(s0-s1)\,(s0-s2)\,(s1-s2)\,\mu r}\right\}\right\}$$

For the r-direction case, we can do a bit of extra work to define matrices when one of the nodes lies on r=0

In[21]:= **MatrixForm[Limit[Mra, s0 → 0] /. r0 → 0]**

Out[21]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{c1^2\,\pi\,r1^2\,(\text{Log}[s1]-\text{Log}[s2])}{d\,(s1-s2)\,\mu r} & \frac{c1\,c2\,\pi\,r1\,r2\,(\text{Log}[s1]-\text{Log}[s2])}{d\,(s1-s2)\,\mu r} \\ 0 & \frac{c1\,c2\,\pi\,r1\,r2\,(\text{Log}[s1]-\text{Log}[s2])}{d\,(s1-s2)\,\mu r} & \frac{c2^2\,\pi\,r2^2\,(\text{Log}[s1]-\text{Log}[s2])}{d\,(s1-s2)\,\mu r} \end{pmatrix}$$

In[22]:= **MatrixForm[Limit[Mra, s1 → 0] /. r1 → 0]**

Out[22]//MatrixForm=

$$\begin{pmatrix} \frac{c0^2\,\pi\,r0^2\,(\text{Log}[s0]-\text{Log}[s2])}{d\,(s0-s2)\,\mu r} & 0 & \frac{c0\,c2\,\pi\,r0\,r2\,(\text{Log}[s0]-\text{Log}[s2])}{d\,(s0-s2)\,\mu r} \\ 0 & 0 & 0 \\ \frac{c0\,c2\,\pi\,r0\,r2\,(\text{Log}[s0]-\text{Log}[s2])}{d\,(s0-s2)\,\mu r} & 0 & \frac{c2^2\,\pi\,r2^2\,(\text{Log}[s0]-\text{Log}[s2])}{d\,(s0-s2)\,\mu r} \end{pmatrix}$$

In[23]:= **MatrixForm[Limit[Mra, s2 → 0] /. r2 → 0]**

Out[23]//MatrixForm=

$$\begin{pmatrix} \frac{c0^2\,\pi\,r0^2\,(\text{Log}[s0]-\text{Log}[s1])}{d\,(s0-s1)\,\mu r} & \frac{c0\,c1\,\pi\,r0\,r1\,(\text{Log}[s0]-\text{Log}[s1])}{d\,(s0-s1)\,\mu r} & 0 \\ \frac{c0\,c1\,\pi\,r0\,r1\,(\text{Log}[s0]-\text{Log}[s1])}{d\,(s0-s1)\,\mu r} & \frac{c1^2\,\pi\,r1^2\,(\text{Log}[s0]-\text{Log}[s1])}{d\,(s0-s1)\,\mu r} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

For the two - nodes - on - zero case, the Mra matrix is all zeros--there’s no stored energy in the R direction because Br=0