Bias Linearization of an 8 - Pole Radial Magnetic Bearing

Angles locating the bearing' s poles

In[1]:=
```
pp = Table[k * 45 Degree, {k, 0, 7}]
```

Out[1]= $\{0, 45\,°, 90\,°, 135\,°, 180\,°, 225\,°, 270\,°, 315\,°\}$

Define some functions to facilitate writing out the even- and odd-harmonic matrices

In[2]:=
```
CC[n_] := Cos[n pp] / Sqrt[Cos[n pp].Cos[n pp]]
```

In[3]:=
```
SS[n_] := Sin[n pp] / Sqrt[Sin[n pp].Sin[n pp]]
```

Define the even- and odd-harmonic matrices

In[4]:=
```
Pe = {CC[2], SS[2], CC[4]}
```

Out[4]=
$$\left\{\left\{\frac{1}{2}, 0, -\frac{1}{2}, 0, \frac{1}{2}, 0, -\frac{1}{2}, 0\right\}, \left\{0, \frac{1}{2}, 0, -\frac{1}{2}, 0, \frac{1}{2}, 0, -\frac{1}{2}\right\}, \left\{\frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right\}\right\}$$

In[5]:=
```
Po = {CC[1], SS[1], CC[3], SS[3]}
```

Out[5]=
$$\left\{\left\{\frac{1}{2}, \frac{1}{2\sqrt{2}}, 0, -\frac{1}{2\sqrt{2}}, -\frac{1}{2}, -\frac{1}{2\sqrt{2}}, 0, \frac{1}{2\sqrt{2}}\right\}, \left\{0, \frac{1}{2\sqrt{2}}, \frac{1}{2}, \frac{1}{2\sqrt{2}}, 0, -\frac{1}{2\sqrt{2}}, -\frac{1}{2}, -\frac{1}{2\sqrt{2}}\right\}, \left\{\frac{1}{2}, -\frac{1}{2\sqrt{2}}, 0, \frac{1}{2\sqrt{2}}, -\frac{1}{2}, \frac{1}{2\sqrt{2}}, 0, -\frac{1}{2\sqrt{2}}\right\}, \left\{0, \frac{1}{2\sqrt{2}}, -\frac{1}{2}, \frac{1}{2\sqrt{2}}, 0, -\frac{1}{2\sqrt{2}}, \frac{1}{2}, -\frac{1}{2\sqrt{2}}\right\}\right\}$$

Define matrix that maps coil currents onto gap flux

In[6]:=
```
B = IdentityMatrix[8] - 1 / 8
```

Out[6]=
$$\left\{\left\{\frac{7}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}\right\}, \left\{-\frac{1}{8}, \frac{7}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}\right\}, \left\{-\frac{1}{8}, -\frac{1}{8}, \frac{7}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}\right\}, \left\{-\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, \frac{7}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}\right\}, \left\{-\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, \frac{7}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}\right\}, \left\{-\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, \frac{7}{8}, -\frac{1}{8}, -\frac{1}{8}\right\}, \left\{-\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, \frac{7}{8}, -\frac{1}{8}\right\}, \left\{-\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}, \frac{7}{8}\right\}\right\}$$

Define matrices that define each pole' s contribution to the various force directions

In[7]:=
```
Dc = DiagonalMatrix[Cos[pp]];
Ds = DiagonalMatrix[Sin[pp]];
```

"Magnetization" matrix is the even harmonic matrix time the flux mapping matrix

In[9]:= **M = Pe.B**

Out[9]= $\left\{\left\{\frac{1}{2}, 0, -\frac{1}{2}, 0, \frac{1}{2}, 0, -\frac{1}{2}, 0\right\}, \left\{0, \frac{1}{2}, 0, -\frac{1}{2}, 0, \frac{1}{2}, 0, -\frac{1}{2}\right\}, \left\{\frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right\}\right\}$

"Field Gradient" matrices are defined as previously derived

In[10]:= **Dx = 2 Pe.Dc.Transpose[Po].Po.B**

Out[10]= $\left\{\{1, 0, 0, 0, -1, 0, 0, 0\}, \left\{0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, -\frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{\sqrt{2}}, \frac{1}{2}, 0, -\frac{1}{2}\right\}\right\}$

In[11]:= **Dy = 2 Pe.Ds.Transpose[Po].Po.B**

Out[11]= $\left\{\{0, 0, -1, 0, 0, 0, 1, 0\}, \left\{0, \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right\}, \left\{0, -\frac{1}{2}, \frac{1}{\sqrt{2}}, -\frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{\sqrt{2}}, \frac{1}{2}\right\}\right\}$

As a check, we can make sure that the rank-reduces forms are exactly the same as the original force-to-current relationships if they are turned back into a symmetric matrices

In[12]:= **MatrixForm[Chop[N[B.Dc.B] - N[(Transpose[M].Dx + Transpose[Dx].M) / 2]]]**

Out[12]//MatrixForm=

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

In[13]:= **MatrixForm[Chop[N[B.Ds.B] - N[(Transpose[M].Dy + Transpose[Dy].M) / 2]]]**

Out[13]//MatrixForm=

$$\begin{pmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{pmatrix}$$

Build up the linear problem to be solved for bias currents in terms of symbolic values for the various magnetization directions

In[14]:= **m = {m1, m2, m3};**

In[15]:= **G = Join[{m.Dx, m.Dy}, M];**

In[16]:= **f = {fx, fy, m1, m2, m3}**

Out[16]= {fx, fy, m1, m2, m3}

In[17]:=
```
Dimensions[G]
```
Out[17]=
```
{5, 8}
```

Compute an analytical solution for i using the Moor - Penrose inverse to solve this underdetermined problem

In[18]:=
```
i = FullSimplify[Transpose[G].Inverse[G.Transpose[G]].f];
```

Bias linearization matrix in analytical form

In[19]:=
```
W = MatrixForm[FullSimplify[Transpose[{(i /. fx → 0 /. fy → 0), D[i, fx], D[i, fy]}]]]
```
Out[19]//MatrixForm=

$$
\begin{pmatrix}
\frac{1}{4}\left(2\,m1+\sqrt{2}\,m3\right) & \frac{\left(2\,m1+\sqrt{2}\,m3\right)\left(m1^2+m2^2-\sqrt{2}\,m1\,m3+m3^2\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} & \frac{m2\,m3\left(\sqrt{2}\,m1+m3\right)}{2\left(\left(m1^2+m2^2\right)^2+m3^4\right)} \\
\frac{1}{4}\left(2\,m2-\sqrt{2}\,m3\right) & \frac{\sqrt{2}\,m2^3+m1\,m2\left(\sqrt{2}\,m1-2\,m3\right)+m2^2\,m3-m3\left(m1^2-\sqrt{2}\,m1\,m3+m3^2\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} & \frac{\sqrt{2}\,m2^3+m2^2\,m3+m1\,m2\left(\sqrt{2}\,m1+2\,m3\right)-m3\left(m1^2+\sqrt{2}\,m1\,m3+m3^2\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} \\
\frac{1}{4}\left(-2\,m1+\sqrt{2}\,m3\right) & \frac{m2\,m3\left(-\sqrt{2}\,m1+m3\right)}{2\left(\left(m1^2+m2^2\right)^2+m3^4\right)} & -\frac{\left(2\,m1-\sqrt{2}\,m3\right)\left(m1^2+m2^2+\sqrt{2}\,m1\,m3+m3^2\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} \\
\frac{1}{4}\left(-2\,m2-\sqrt{2}\,m3\right) & \frac{\sqrt{2}\,m2^3-m2^2\,m3+m3^3+m1^2\left(\sqrt{2}\,m2+m3\right)-m1\,m3\left(2\,m2+\sqrt{2}\,m3\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} & -\frac{\sqrt{2}\,m2^3-m2^2\,m3+m3^3+m1^2\left(\sqrt{2}\,m2+m3\right)+m1\,m3\left(2\,m2+\sqrt{2}\,m3\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} \\
\frac{1}{4}\left(2\,m1+\sqrt{2}\,m3\right) & -\frac{\left(2\,m1+\sqrt{2}\,m3\right)\left(m1^2+m2^2-\sqrt{2}\,m1\,m3+m3^2\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} & -\frac{m2\,m3\left(\sqrt{2}\,m1+m3\right)}{2\left(\left(m1^2+m2^2\right)^2+m3^4\right)} \\
\frac{1}{4}\left(2\,m2-\sqrt{2}\,m3\right) & \frac{-\sqrt{2}\,m2^3-m2^2\,m3+m3^3+m1^2\left(-\sqrt{2}\,m2+m3\right)+m1\,m3\left(2\,m2-\sqrt{2}\,m3\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} & \frac{-\sqrt{2}\,m2^3-m2^2\,m3+m3^3+m1^2\left(-\sqrt{2}\,m2+m3\right)+m1\,m3\left(-2\,m2+\sqrt{2}\,m3\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} \\
\frac{1}{4}\left(-2\,m1+\sqrt{2}\,m3\right) & \frac{m2\left(\sqrt{2}\,m1-m3\right)m3}{2\left(\left(m1^2+m2^2\right)^2+m3^4\right)} & \frac{\left(2\,m1-\sqrt{2}\,m3\right)\left(m1^2+m2^2+\sqrt{2}\,m1\,m3+m3^2\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} \\
\frac{1}{4}\left(-2\,m2-\sqrt{2}\,m3\right) & -\frac{\sqrt{2}\,m2^3-m2^2\,m3+m3^3+m1^2\left(\sqrt{2}\,m2+m3\right)-m1\,m3\left(2\,m2+\sqrt{2}\,m3\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)} & \frac{\sqrt{2}\,m2^3-m2^2\,m3+m3^3+m1^2\left(\sqrt{2}\,m2+m3\right)+m1\,m3\left(2\,m2+\sqrt{2}\,m3\right)}{4\left(\left(m1^2+m2^2\right)^2+m3^4\right)}
\end{pmatrix}
$$

NNSS - type solution

In[20]:=
```
MatrixForm[N[i] /. {m1 → 1/Sqrt[2], m2 → 1/Sqrt[2], m3 → 0}]
```
Out[20]//MatrixForm=

$$
\begin{pmatrix}
0.353553\,(1.+1.\,fx) \\
0.25\,(1.41421+1.\,fx+1.\,fy) \\
-0.353553\,(1.+1.\,fy) \\
-0.25\,(1.41421-1.\,fx+1.\,fy) \\
0.353553\,(1.-1.\,fx) \\
0.25\,(1.41421-1.\,fx-1.\,fy) \\
-0.353553\,(1.-1.\,fy) \\
-0.25\,(1.41421+1.\,fx-1.\,fy)
\end{pmatrix}
$$

NSNS - type solution

In[21]:= `MatrixForm[N[i] /. {m1 → 0 , m2 → 0, m3 → 1}]`

Out[21]//MatrixForm=

$$\begin{pmatrix} 0.353553\ (1. + 1.\ fx) \\ 0.25\ (-1.41421 - 1.\ fx - 1.\ fy) \\ 0.353553\ (1. + 1.\ fy) \\ -0.25\ (1.41421 - 1.\ fx + 1.\ fy) \\ 0.353553\ (1. - 1.\ fx) \\ 0.25\ (-1.41421 + 1.\ fx + 1.\ fy) \\ 0.353553\ (1. - 1.\ fy) \\ -0.25\ (1.41421 + 1.\ fx - 1.\ fy) \end{pmatrix}$$