MORE GENERALLY...

$P_c \rightarrow$ COLUMNS ARE ORTHONORMAL BASIS OF A JOINT INNER PRODUCT NULL SPACE

$P_b \rightarrow$ ORTHONORMAL BASIS OF EVERYTHING $\perp$ TO P_c

LET $P = [P_b \; P_c]$; $P'P = I$

IN SOME DIRECTION,

$$F = i' M i \quad \text{WHERE } M \text{ IS SYMMETRIC, INDEF.}$$

$$= i' P P' M P P' i$$

$$= u' P' M P u \quad \text{WHERE } u = P' i$$

$\rightarrow$ EXPAND $P'MP$

$$P'MP = \begin{bmatrix} P_b' \\ P_c' \end{bmatrix} M \begin{bmatrix} P_b & P_c \end{bmatrix}$$

$$= \begin{bmatrix} P_b' M P_b & P_b' M P_c \\ P_c' M P_b & P_c' M P_c \end{bmatrix}$$

→ CAN ADD ARBITRARY SKEW-SYMMETRIC MATRIX TO $P'MP$ W/OUT CHANGING INNER PRODUCTS COMPUTED WITH IT

→ SELECT THE SKEW-SYMMETRIC MATRIX

$$\begin{bmatrix} 0 & P_b' M P_c \\ -P_c' M P_b & 0 \end{bmatrix} = S$$

THEN,

$$P'MU + S = \begin{bmatrix} P_b' M P_b & 2P_b' M P_c \\ 0 & 0 \end{bmatrix}$$

WHERE LOWER-RIGHT CORNER IS ZERO BY DEFINITION OF P_c.

NOW,

$$F = U' \begin{bmatrix} P_b' M P_b & 2P_b' M P_c \\ 0 & 0 \end{bmatrix} U$$

$$F = i'(P_b P_b' M P_b P_b' + 2 P_b P_b' M P_c P_c') i$$

CAN THEN WRITE:

$$\boxed{\begin{aligned} m'\left(P_b' M_1 P_b P_b' + 2 P_b M_1 P_c' P_c\right) i &= F_1 \\ m'\left(P_b' M_2 P_b P_b' + 2 P_b M_2 P_c' P_c\right) i &= F_2 \\ P_b' i &= m \end{aligned}} \tag{1}$$

→ m SELECTED "ARBITRARILY" AND PARAMETERIZES THE SOLUTIONS

REWRITE (1) FOR CONVENIENCE AS:

$$G i = f$$

AND LET Q BE A PD WEIGHTING MATRIX

$$i = Q G' (G Q G')^{-1} f$$

CAN THEN REACH AN ENTIRE MANIFOLD OF SOLUTIONS INDEXED BY CHOICE OF m & Q.

→ ACCOMMODATE FAULTS BY ADDING EXTRA CONSTRAINT EQS TO (1)

FOR EVEN-POLE BEARINGS W/ REGULAR SPACING, CAN WRITE DOWN VALID CHOICES OF P_c BY INSPECTION;

Θ = LIST OF POLE ANGLES

LET $C_k = \cos(k\Theta) / \sqrt{\cos(k\Theta)\cos(k\Theta)'}$

$S_k = \sin(k\Theta) / \sqrt{\sin(k\Theta)\sin(k\Theta)'}$

CHOICES FOR P_c FOR 8-POLE BEARING:

$\left[C_0 \mid C_2 \mid S_2 \mid C_4 \right]$ → CONTAINS "CLASSIC" NNSS, NSNS SCHEMES

$\left[C_0 \mid C_1 \mid S_1 \mid C_3 \mid S_3 \right]$

$\left[C_0 \mid C_1 \mid S_1 \mid C_4 \right]$

WHICH WE GET BY INSPECTION FROM "ORTHOGONALITY OF DFT SINES & COSINES" RESULT