

```
<< c:\femm42\mathfemm\mathfemm.m
```

```
MathFEMM loaded at Mon 18 Sep 2017 08:42:56
```

```
OpenFEMM[];
```

```
In[1]:= Ia = 4; (* Phase current amplitude *)
```

```
β = -30 * Degree; (* Phase advance *)
```

```
θ = 0; (* Rotor angle *)
```

```
p = 3; (* Number of rotor pole pairs *)
```

```
dθ = 2 / 3;
```

```
In[6]:= Idq = {-Ia * Sin[β], Ia * Cos[β]};
```

```
tto = {}; (* Array for storing the total torque,  
with the full current and PMs turned on *)
```

```
ψfpm = {}; (* DQ flux linkage for FP case  
where the PMs are on but the currents are off *)
```

```
ψfpd = {}; (* DQ flux linkage for FP case where  
the PMs are off and only Id is on *)
```

```
ψfpq = {}; (* DQ flux linkage for FP case where  
the PMs are off and only Iq is on *)
```

```
tta = {}; (* keep track of angle associated with each data item *)
```

```
SmartMesh[1];
```

```
For[θ = 0, θ ≤ 40, θ += dθ,
```

```
  tta = Append[tta, θ];
```

```
  OpenDocument[NotebookDirectory[] <> "zhu.fem"];
```

```
  MISaveAs[NotebookDirectory[] <> "temp.fem"];
```

```
  MISelectGroup[1];
```

```
  MIMoveRotate[0, 0, θ, 4];
```

```
  (* DQ to ABC, phase current amplitude scaling *)
```

```
  K = Transpose[{  
    {Sin[p * θ * Degree], Sin[(p * θ + 120) * Degree], Sin[(p * θ - 120) * Degree]},  
    {Cos[p * θ * Degree], Cos[(p * θ + 120) * Degree], Cos[(p * θ - 120) * Degree]}}];
```

```
  (* ABC to DQ *)
```

```
  Kt = (2 / 3) * Transpose[K];
```

```
  (* Full Solution *)
```

```
  Iabc = K.Idq;
```

```
  MISetCurrent["A", Iabc[[1]]];
```

```
  MISetCurrent["B", Iabc[[2]]];
```

```
  MISetCurrent["C", Iabc[[3]]];
```

```
  MIModifyMaterial["NdFeB", 3, 891 000];
```

```
  MIAnalyze[];
```

```
  MISelectGroup[1];
```

```
  MIMoveRotate[0, 0, dθ, 4];
```

```
  MIClearSelected[];
```

```
  MIloadSolution[];
```

```
  MOGroupSelectBlock[1];
```

```
  tt = MOBlockIntegral[22];
```

```
  tto = Append[tto, tt];
```

```
  MOClose[];
```

```

Print[N[ $\theta$ ], " ", tt];

(* Magnet Only Solution *)
Iabc = K.{0, 0};
MISetCurrent["A", Iabc[[1]]];
MISetCurrent["B", Iabc[[2]]];
MISetCurrent["C", Iabc[[3]]];
MIModifyMaterial["NdFeB", 3, 891 000];
MISetPrevious["temp.ans", 2];
MISaveAs[NotebookDirectory[] <> "mTemp.fem"];
MIAnalyze[];
MILoadSolution[];
 $\psi_{dq}$  = Kt.{MOGetCircuitProperties["A"][[3]],
    MOGetCircuitProperties["B"][[3]], MOGetCircuitProperties["C"][[3]]};
 $\psi_{fpm}$  = Append[ $\psi_{fpm}$ ,  $\psi_{dq}$ ];
MOClose[];

(* D-Axis Only Solution *)
Iabc = K.{Idq[[1]], 0};
MISetCurrent["A", Iabc[[1]]];
MISetCurrent["B", Iabc[[2]]];
MISetCurrent["C", Iabc[[3]]];
MIModifyMaterial["NdFeB", 3, 0];
MISetPrevious["temp.ans", 2];
MISaveAs[NotebookDirectory[] <> "dTemp.fem"];
MIAnalyze[];
MILoadSolution[];
 $\psi_{dq}$  = Kt.{MOGetCircuitProperties["A"][[3]],
    MOGetCircuitProperties["B"][[3]], MOGetCircuitProperties["C"][[3]]};
 $\psi_{fpd}$  = Append[ $\psi_{fpd}$ ,  $\psi_{dq}$ ];
MOClose[];

(* Q-Axis Only Solution *)
Iabc = K.{0, Idq[[2]]};
MISetCurrent["A", Iabc[[1]]];
MISetCurrent["B", Iabc[[2]]];
MISetCurrent["C", Iabc[[3]]];
MIModifyMaterial["NdFeB", 3, 0];
MISetPrevious["temp.ans", 2];
MISaveAs[NotebookDirectory[] <> "qTemp.fem"];
MIAnalyze[];
MILoadSolution[];
 $\psi_{dq}$  = Kt.{MOGetCircuitProperties["A"][[3]],
    MOGetCircuitProperties["B"][[3]], MOGetCircuitProperties["C"][[3]]};
 $\psi_{fpq}$  = Append[ $\psi_{fpq}$ ,  $\psi_{dq}$ ];
MOClose[];

MIClose[];
];
CloseFEMM[]

```

Print out results of FEA so we could make other plots later without re - running the analysis

In[64]:= **tto**

Out[64]= {2.15098, 1.8747, 1.82046, 1.90602, 2.07827, 2.27823, 2.41448, 2.54103, 2.66856,
2.76972, 2.85858, 2.9624, 3.07315, 3.15684, 3.27632, 3.34475, 3.43753, 3.55071,
3.61939, 3.67956, 3.70873, 3.76602, 3.78503, 3.77982, 3.77891, 3.72372,
3.62009, 3.43022, 3.07605, 2.60468, 2.15113, 1.87367, 1.8197, 1.90402, 2.07565,
2.2786, 2.41888, 2.54141, 2.67357, 2.76755, 2.85937, 2.96383, 3.07442, 3.15681,
3.27403, 3.35299, 3.44393, 3.54206, 3.61919, 3.67963, 3.72099, 3.76406, 3.78958,
3.78142, 3.77941, 3.72507, 3.61994, 3.43029, 3.07666, 2.60366, 2.15144}

In[65]:= **ψ fpm**

Out[65]= {{0.24993, -0.0283573}, {0.249399, -0.0308708},
{0.248252, -0.032535}, {0.246412, -0.0331372}, {0.243863, -0.032601},
{0.240938, -0.0312985}, {0.237957, -0.0296427},
{0.23525, -0.0279319}, {0.232907, -0.0262583}, {0.230872, -0.0245847},
{0.229239, -0.0230388}, {0.228001, -0.0215607}, {0.227037, -0.0201302},
{0.226397, -0.0187135}, {0.226109, -0.0173134}, {0.226017, -0.0160146},
{0.226133, -0.0145823}, {0.226662, -0.0136379}, {0.22755, -0.0128113},
{0.228704, -0.0121492}, {0.230202, -0.0117453}, {0.232117, -0.0116064},
{0.234298, -0.0117939}, {0.236749, -0.0123992}, {0.239512, -0.0133803},
{0.242355, -0.0147824}, {0.245031, -0.0167874}, {0.247297, -0.0193196},
{0.248901, -0.0222271}, {0.24977, -0.0253521}, {0.249935, -0.0283548},
{0.249411, -0.0308558}, {0.248242, -0.0325098}, {0.246427, -0.0331493},
{0.243895, -0.0326111}, {0.240918, -0.0312737}, {0.23797, -0.0296524},
{0.235245, -0.0279237}, {0.232908, -0.0262618}, {0.23087, -0.0245798},
{0.229242, -0.0230378}, {0.228008, -0.0215641}, {0.227049, -0.0201164},
{0.226397, -0.0186981}, {0.226117, -0.0173201}, {0.226007, -0.0160266},
{0.226117, -0.0145675}, {0.226672, -0.013643}, {0.227546, -0.0128235},
{0.228722, -0.0121366}, {0.230219, -0.0117521}, {0.232118, -0.0116002},
{0.234303, -0.0117842}, {0.236757, -0.0124037}, {0.239509, -0.0133811},
{0.242342, -0.0148025}, {0.245032, -0.0167915}, {0.247303, -0.0193125},
{0.248904, -0.0222238}, {0.249774, -0.025354}, {0.249942, -0.0283438}}

In[66]:= ψ_{fpd}

```
Out[66]= {{0.0413763, -0.00480418}, {0.0414802, -0.00368116},
  {0.041932, -0.0026607}, {0.0427283, -0.00182738}, {0.0438149, -0.00121301},
  {0.0451031, -0.000844588}, {0.0464922, -0.000717783},
  {0.0478418, -0.000783256}, {0.0490908, -0.00100688}, {0.0502721, -0.00140765},
  {0.0512885, -0.00190277}, {0.0521528, -0.00250926}, {0.0528812, -0.00320531},
  {0.0534125, -0.00394287}, {0.053737, -0.0047015}, {0.0538468, -0.00540826},
  {0.0537334, -0.00606042}, {0.0534703, -0.0067592}, {0.0529859, -0.00739141},
  {0.0523549, -0.00791407}, {0.0515409, -0.00832513}, {0.0505674, -0.00865378},
  {0.049478, -0.00886548}, {0.0482702, -0.00894579}, {0.0469302, -0.00891497},
  {0.0455522, -0.00873999}, {0.0442327, -0.00835312}, {0.0430718, -0.0077468},
  {0.0421889, -0.00692715}, {0.0416204, -0.00592067}, {0.0413734, -0.00480376},
  {0.0414783, -0.00367938}, {0.0419401, -0.00266632}, {0.0427191, -0.00182192},
  {0.0438057, -0.00120432}, {0.0451149, -0.000850898}, {0.0464832, -0.000714421},
  {0.0478459, -0.00078369}, {0.0490934, -0.00100649}, {0.0502738, -0.00140775},
  {0.0512909, -0.00190363}, {0.0521521, -0.00250889}, {0.05288, -0.00320394},
  {0.0534159, -0.00394706}, {0.0537358, -0.00469807}, {0.0538525, -0.00540608},
  {0.0537396, -0.00606264}, {0.0534676, -0.00675798}, {0.052992, -0.0073891},
  {0.0523513, -0.00791768}, {0.0515389, -0.0083256}, {0.0505689, -0.00865574},
  {0.0494773, -0.0088677}, {0.0482712, -0.00894616}, {0.0469359, -0.00891403},
  {0.045558, -0.00873356}, {0.0442347, -0.0083518}, {0.0430771, -0.00774726},
  {0.0421883, -0.00692691}, {0.041622, -0.0059221}, {0.0413765, -0.00480197}}
```

In[67]:= ψ_{fpq}

```
Out[67]= {{-0.00832108, 0.181088}, {-0.00637596, 0.179576},
  {-0.00460846, 0.177294}, {-0.00316512, 0.174457},
  {-0.002101, 0.171318}, {-0.00146287, 0.16822}, {-0.00124324, 0.165418},
  {-0.00135664, 0.162923}, {-0.00174397, 0.160779}, {-0.00243812, 0.15911},
  {-0.00329569, 0.157834}, {-0.00434616, 0.156979}, {-0.00555176, 0.156629},
  {-0.00682924, 0.156699}, {-0.00814323, 0.157112}, {-0.00936739, 0.157891},
  {-0.010497, 0.158847}, {-0.0117073, 0.16047}, {-0.0128023, 0.162209},
  {-0.0137076, 0.164017}, {-0.0144196, 0.165874}, {-0.0149888, 0.167798},
  {-0.0153555, 0.169815}, {-0.0154946, 0.171878}, {-0.0154412, 0.173909},
  {-0.0151381, 0.175985}, {-0.014468, 0.17799}, {-0.0134179, 0.179775},
  {-0.0119982, 0.181131}, {-0.0102549, 0.18164}, {-0.00832036, 0.181087},
  {-0.00637287, 0.17958}, {-0.00461821, 0.177311}, {-0.00315565, 0.174451},
  {-0.00208594, 0.171312}, {-0.0014738, 0.168246}, {-0.00123741, 0.165413},
  {-0.00135739, 0.16293}, {-0.00174329, 0.160787}, {-0.0024383, 0.159115},
  {-0.00329718, 0.15783}, {-0.00434553, 0.156972}, {-0.00554938, 0.156618},
  {-0.00683651, 0.156704}, {-0.00813729, 0.157103}, {-0.00936361, 0.15791},
  {-0.0105008, 0.158869}, {-0.0117052, 0.160457}, {-0.0127983, 0.162216},
  {-0.0137138, 0.164006}, {-0.0144204, 0.165863}, {-0.0149922, 0.167815},
  {-0.0153593, 0.169821}, {-0.0154952, 0.17188}, {-0.0154396, 0.173927},
  {-0.015127, 0.175997}, {-0.0144657, 0.177997}, {-0.0134186, 0.179787},
  {-0.0119978, 0.18112}, {-0.0102574, 0.181641}, {-0.00831725, 0.181086}}
```

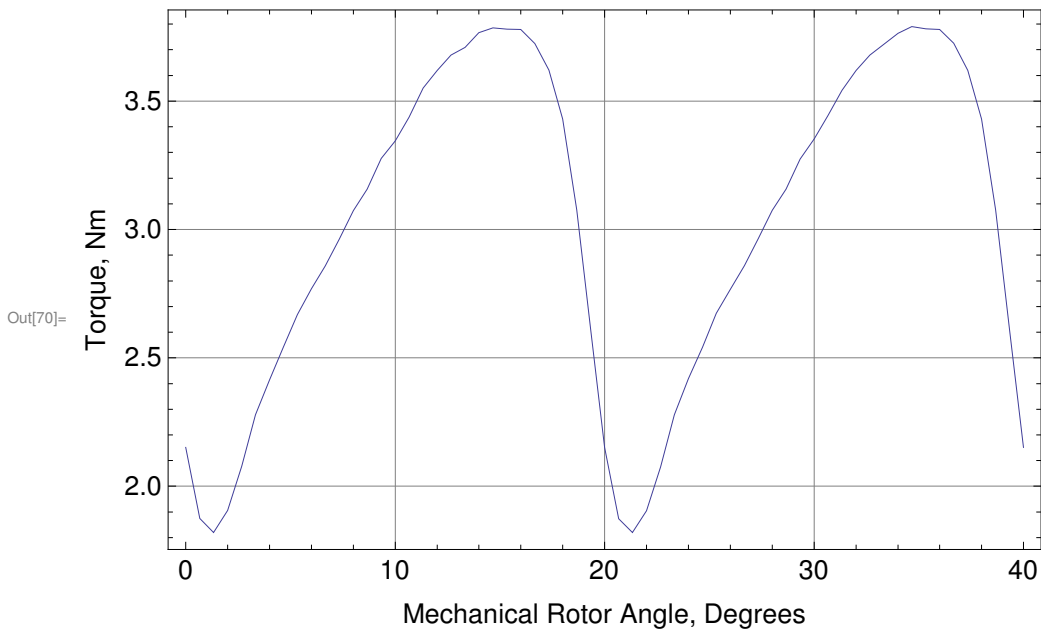
In[68]:= **N[tta]**

Out[68]= {0., 0.666667, 1.33333, 2., 2.66667, 3.33333, 4., 4.66667, 5.33333, 6., 6.66667, 7.33333, 8., 8.66667, 9.33333, 10., 10.6667, 11.3333, 12., 12.6667, 13.3333, 14., 14.6667, 15.3333, 16., 16.6667, 17.3333, 18., 18.6667, 19.3333, 20., 20.6667, 21.3333, 22., 22.6667, 23.3333, 24., 24.6667, 25.3333, 26., 26.6667, 27.3333, 28., 28.6667, 29.3333, 30., 30.6667, 31.3333, 32., 32.6667, 33.3333, 34., 34.6667, 35.3333, 36., 36.6667, 37.3333, 38., 38.6667, 39.3333, 40.}

In[69]:= **X[z_] := Transpose[{tta, z}]**

Plot of total flux from FEA

In[70]:= **ListPlot[X[tto], Joined → True, PlotRange → All, Frame → True, GridLines → Automatic, ImageSize → 500, FrameLabel → {"Mechanical Rotor Angle, Degrees", "Torque, Nm"}, BaseStyle → {FontFamily → "Arial", FontSize → 14}]**



In[71]:= **Z = {{0, p}, {-p, 0}} /. p → 3;**

"Average" PM portion of the torque. Here, we don't consider spatial variations of the flux linkage, so it doesn't get the variations with angle quite right.

In[72]:= **t_{pm} = (3 / 2) * ψ_{fpm}.Z.Idq**

Out[72]= {4.15124, 4.16558, 4.16269, 4.13942, 4.09485, 4.03754, 3.97617, 3.91857, 3.86699, 3.8202, 3.78083, 3.74823, 3.72032, 3.6976, 3.68051, 3.66738, 3.6563, 3.65606, 3.66245, 3.67449, 3.69421, 3.7228, 3.75849, 3.80215, 3.85404, 3.91098, 3.97074, 4.02886, 4.08002, 4.1217, 4.1513, 4.16563, 4.1623, 4.13976, 4.09544, 4.03701, 3.97645, 3.91843, 3.86703, 3.82013, 3.78087, 3.74837, 3.72039, 3.69747, 3.6807, 3.66733, 3.65592, 3.65626, 3.6625, 3.67466, 3.69453, 3.72276, 3.75847, 3.80232, 3.85401, 3.91097, 3.9708, 4.02888, 4.08004, 4.12178, 4.1513}

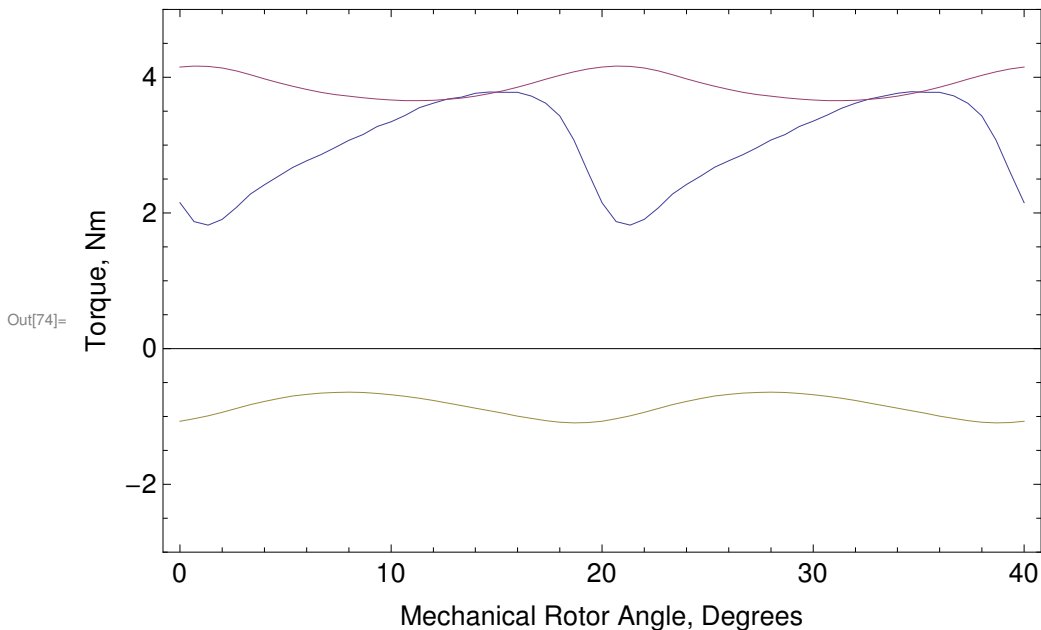
"Average" reluctance portion of the torque. Here, we don't consider spatial variations of the flux linkage, so it doesn't get the variations with angle quite right.

```
In[73]:= tr1 = (3 / 2) * ( $\psi_{fpd}$  +  $\psi_{fpq}$ ) . Z . Idq
```

```
Out[73]:= {-1.07128, -1.03583, -0.989884, -0.936937, -0.880691, -0.8261, -0.776937,
-0.73463, -0.69988, -0.673664, -0.655246, -0.645, -0.643021, -0.648645,
-0.660956, -0.678983, -0.701087, -0.732376, -0.766957, -0.802475, -0.839271,
-0.877685, -0.916624, -0.955471, -0.994086, -1.0311, -1.06275, -1.08599,
-1.09721, -1.09254, -1.0713, -1.03587, -0.990008, -0.936931, -0.880625,
-0.826256, -0.776972, -0.734632, -0.699906, -0.67368, -0.65519, -0.644936,
-0.642914, -0.648711, -0.660838, -0.679021, -0.701228, -0.732279,
-0.766885, -0.802493, -0.839213, -0.877845, -0.916736, -0.955474,
-0.99414, -1.031, -1.06276, -1.08603, -1.09712, -1.09254, -1.07121}
```

Plot of total torque and the "average" torque decomposition

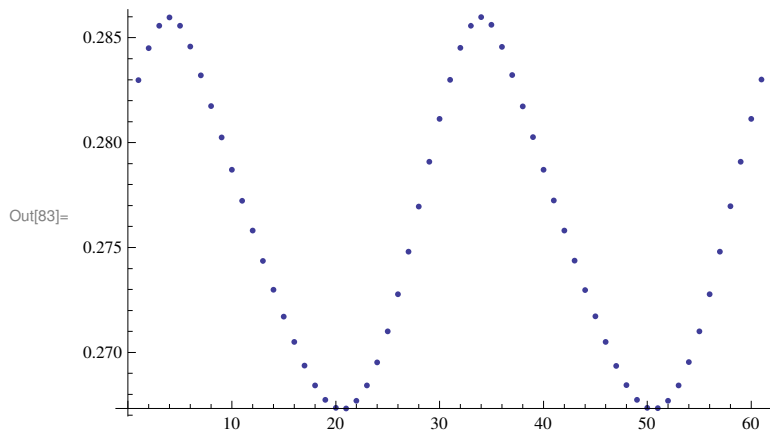
```
In[74]:= ListPlot[{X[tto], X[tpm], X[tr1]}, Joined -> True, Frame -> True, PlotRange -> {-3, 5},
ImageSize -> 500, FrameLabel -> {"Mechanical Rotor Angle, Degrees", "Torque, Nm"},
BaseStyle -> {FontFamily -> "Arial", FontSize -> 14}]
```



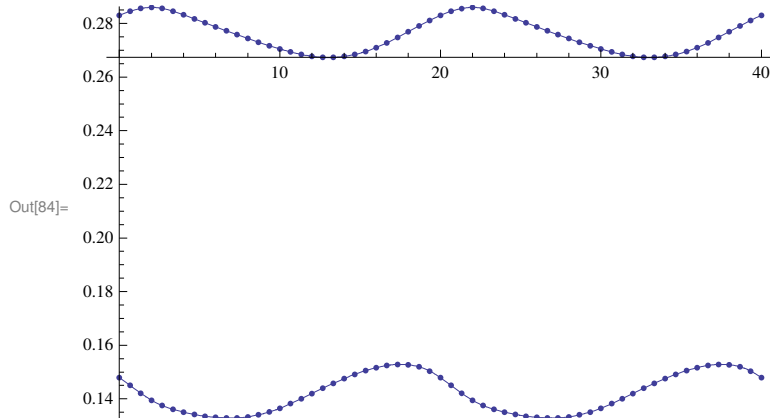
Compute change in flux linkage with change in angle. Needed to compute the input torque

```
In[75]:=  $\psi_{dq}$  =  $\psi_{fpm}$  +  $\psi_{fpd}$  +  $\psi_{fpq}$ ;
fpdq1 = Transpose[{tta,  $\psi_{dq}$ .{1, 0}}];
ffpdq1 = Fit[fpdq1, Join[Table[Cos[k * 2 Pi x / 20], {k, 0, 10}],
Table[Sin[k * 2 Pi x / 20], {k, 0, 10}]], x];
d $\psi_{fpdq1}$  = D[ffpdq1, x] / Degree /. x -> tta;
fpdq2 = Transpose[{tta,  $\psi_{dq}$ .{0, 1}}];
ffpdq2 = Fit[fpdq2, Join[Table[Cos[k * 2 Pi x / 20], {k, 0, 10}],
Table[Sin[k * 2 Pi x / 20], {k, 0, 10}]], x];
d $\psi_{fpdq2}$  = D[ffpdq2, x] / Degree /. x -> tta;
d $\psi_{dq}$  = Transpose[{d $\psi_{fpdq1}$ , d $\psi_{fpdq2}$ }]
```

In[83]:= **ListPlot**[ψ_{dq} .{1, 0}]



In[84]:= **Show**[**ListPlot**[f_{pdq1}], **Plot**[f_{fpdq1}, {x, 0, 40}],
ListPlot[f_{pdq2}], **Plot**[f_{fpdq2}, {x, 0, 40}], **PlotRange** → **All**]



Compute "input torque" due to power actually supplied at the terminals of the machine

In[85]:= **t_{in}** = (3 / 2) * (ψ_{dq} .Z.Id_q + d ψ_{dq} .Id_q)

Out[85]= {2.29737, 2.14272, 2.10449, 2.16464, 2.27548, 2.36435, 2.41269, 2.45446, 2.52956,
2.61915, 2.69753, 2.8153, 2.97503, 3.10486, 3.23535, 3.39412, 3.49822,
3.55791, 3.62981, 3.67131, 3.69699, 3.74669, 3.75261, 3.72459, 3.72032, 3.66,
3.5121, 3.31987, 3.01645, 2.61416, 2.2974, 2.14274, 2.10398, 2.16499, 2.27614,
2.36367, 2.41293, 2.45431, 2.52957, 2.61907, 2.69763, 2.8155, 2.9752, 3.10466,
3.23565, 3.39404, 3.4977, 3.55821, 3.62993, 3.67146, 3.69737, 3.74648, 3.75248,
3.72475, 3.72023, 3.66009, 3.51215, 3.31986, 3.01656, 2.61423, 2.29749}

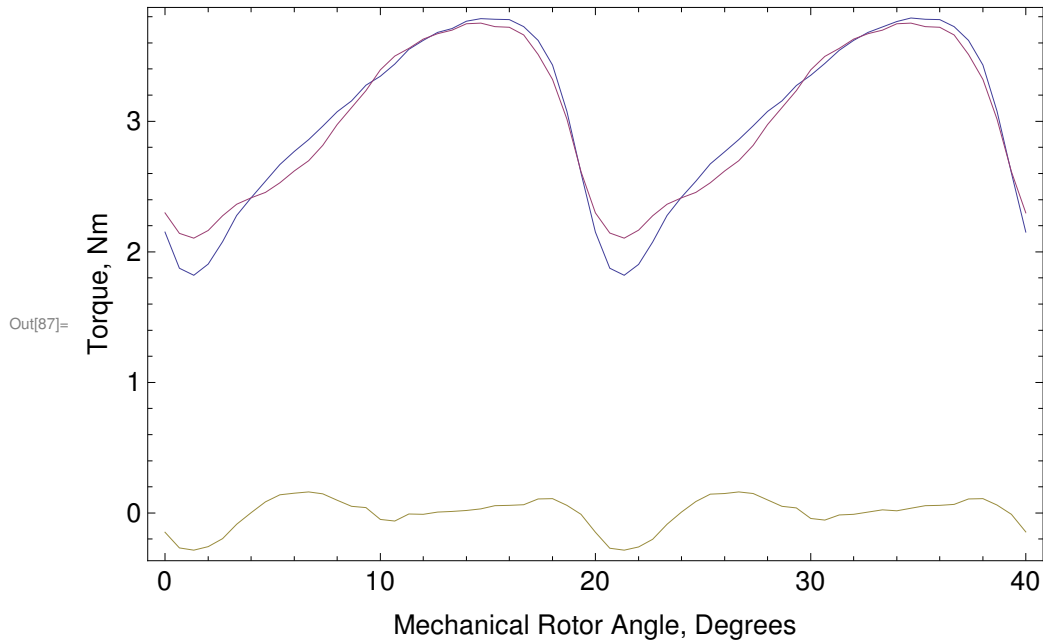
Cogging torque is the left-over part of the total torque, due to variations in stored energy in the magnetic field as the rotor spins

In[86]:= **tcog = tto - tin**

Out[86]= {-0.146388, -0.268026, -0.284026, -0.25862, -0.197208, -0.0861249, 0.00179673, 0.0865707, 0.138997, 0.150569, 0.161047, 0.1471, 0.0981236, 0.0519845, 0.0409727, -0.0493688, -0.0606853, -0.00720264, -0.0104197, 0.00825372, 0.0117396, 0.0193317, 0.0324169, 0.0552369, 0.05859, 0.0637171, 0.107991, 0.110343, 0.0595991, -0.00947543, -0.146269, -0.269074, -0.284278, -0.260964, -0.200486, -0.0850691, 0.00594523, 0.0870915, 0.143997, 0.148481, 0.161741, 0.148327, 0.0992145, 0.0521546, 0.0383791, -0.0410438, -0.0537627, -0.0161473, -0.010743, 0.00817196, 0.0236172, 0.0175808, 0.0370954, 0.0566689, 0.0591764, 0.0649792, 0.10779, 0.11043, 0.0601032, -0.010567, -0.146056}

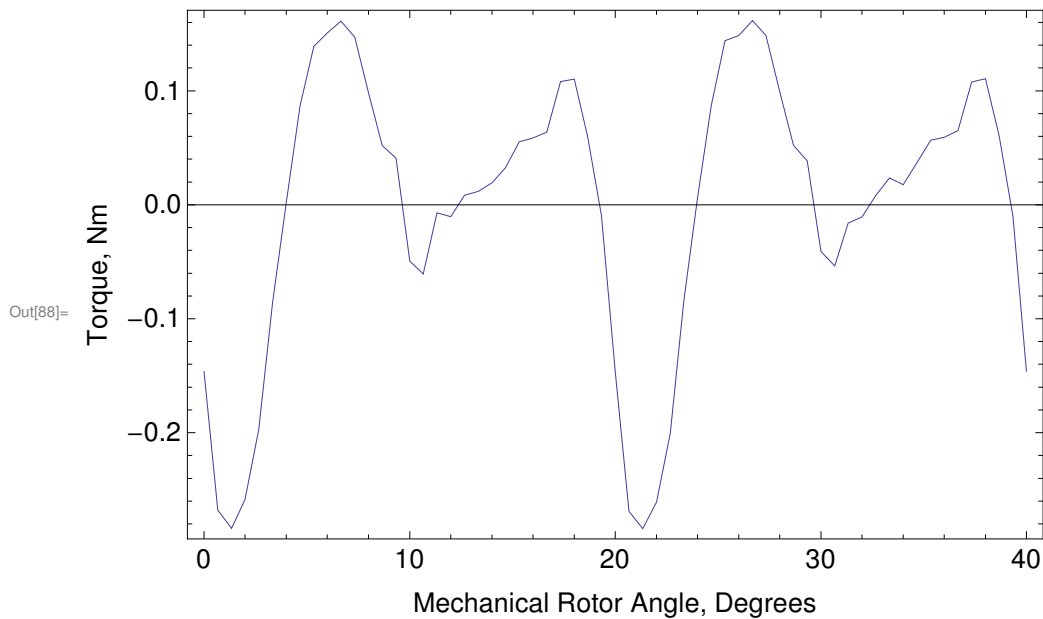
Plot of total torque, input torque, and cogging torque

In[87]:= **ListPlot[{X[tto], X[tin], X[tcog]}, Joined → True, Frame → True, ImageSize → 500, FrameLabel → {"Mechanical Rotor Angle, Degrees", "Torque, Nm"}, BaseStyle → {FontFamily → "Arial", FontSize → 14}]**



Plot of just the cogging torque


```
In[88]:= ListPlot[X[tcog], Joined → True, Frame → True, ImageSize → 500,
  FrameLabel → {"Mechanical Rotor Angle, Degrees", "Torque, Nm"},
  BaseStyle → {FontFamily → "Arial", FontSize → 14}]
```



Now, we can take derivatives of components of the flux to get torque separations based on flux linkage that include accurate variations of torque with position

Portion of the computation of PM torque

```
In[89]:=  $\psi_{dq} = \psi_{fpm} + 0 * \psi_{fpd} + 0 * \psi_{fpq};$ 
fpdq1 = Transpose[{tta,  $\psi_{dq}$ .{1, 0}}];
ffpdq1 = Fit[fpdq1, Join[Table[Cos[k * 2 Pi x / 20], {k, 0, 10}],
  Table[Sin[k * 2 Pi x / 20], {k, 0, 10}]], x];
d $\psi$ /fpdq1 = D[ffpdq1, x] / Degree /. x → tta;
fpdq2 = Transpose[{tta,  $\psi_{dq}$ .{0, 1}}];
ffpdq2 = Fit[fpdq2, Join[Table[Cos[k * 2 Pi x / 20], {k, 0, 10}],
  Table[Sin[k * 2 Pi x / 20], {k, 0, 10}]], x];
d $\psi$ /fpdq2 = D[ffpdq2, x] / Degree /. x → tta;
d $\psi$ /dqm = Transpose[{d $\psi$ /fpdq1, d $\psi$ /fpdq2}];
```

Portion of the computation of reluctance torque

```
In[97]:=  $\psi_{dq} = 0 * \psi_{fpm} + \psi_{fpd} + \psi_{fpq};$ 
fpdq1 = Transpose[{tta,  $\psi_{dq}$ .{1, 0}}];
ffpdq1 = Fit[fpdq1, Join[Table[Cos[k * 2 Pi x / 20], {k, 0, 10}],
  Table[Sin[k * 2 Pi x / 20], {k, 0, 10}]], x];
d $\psi$ /fpdq1 = D[ffpdq1, x] / Degree /. x → tta;
fpdq2 = Transpose[{tta,  $\psi_{dq}$ .{0, 1}}];
ffpdq2 = Fit[fpdq2, Join[Table[Cos[k * 2 Pi x / 20], {k, 0, 10}],
  Table[Sin[k * 2 Pi x / 20], {k, 0, 10}]], x];
d $\psi$ /fpdq2 = D[ffpdq2, x] / Degree /. x → tta;
d $\psi$ /dqr = Transpose[{d $\psi$ /fpdq1, d $\psi$ /fpdq2}];
```

Complete PM torque component derived via flux linkage

```
In[105]:= tfpm = (3 / 2) * (ψfpm.Z.Idq + dψ/dqm.Idq)
```

```
Out[105]:= {2.84814, 2.99734, 3.24909, 3.56983, 3.82005, 3.9444, 4.0024, 4.02387, 4.04624,
  4.07677, 4.08127, 4.1073, 4.16612, 4.19274, 4.234, 4.29589, 4.27164, 4.23002,
  4.25562, 4.26655, 4.2541, 4.24785, 4.18531, 4.11365, 4.06174, 3.88659, 3.59349,
  3.31012, 3.04064, 2.84863, 2.8482, 2.9974, 3.24871, 3.57017, 3.82064, 3.94387,
  4.00268, 4.02372, 4.04628, 4.07671, 4.08132, 4.10744, 4.16619, 4.19261,
  4.23419, 4.29585, 4.27126, 4.23023, 4.25567, 4.26672, 4.25443, 4.2478, 4.18529,
  4.11381, 4.06171, 3.88657, 3.59355, 3.31014, 3.04066, 2.84871, 2.8482}
```

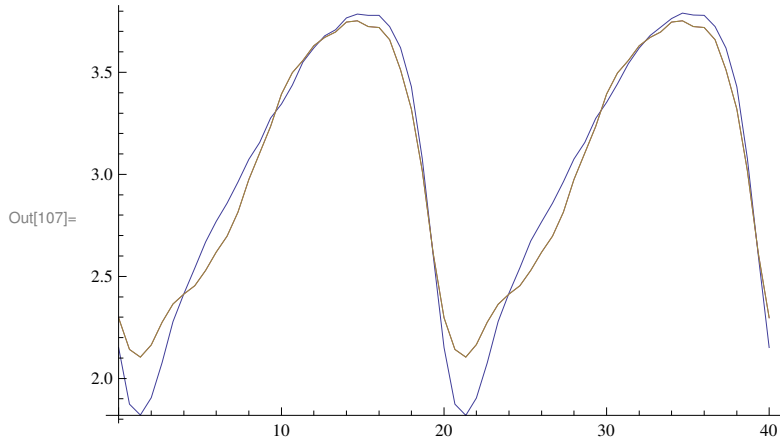
Complete reluctance torque component derived via flux linkage

```
In[106]:= tfpr = (3 / 2) * ((ψfpd + ψfpq).Z.Idq + dψ/dqr.Idq)
```

```
Out[106]:= {-0.550773, -0.854621, -1.1446, -1.40519, -1.54457, -1.58005, -1.58971,
  -1.5694, -1.51668, -1.45762, -1.38374, -1.292, -1.19109, -1.08789,
  -0.998656, -0.901771, -0.773421, -0.672112, -0.625809, -0.595244,
  -0.557113, -0.501162, -0.4327, -0.389057, -0.341425, -0.226586, -0.081388,
  0.00975222, -0.0241878, -0.234476, -0.550797, -0.854655, -1.14473, -1.40518,
  -1.5445, -1.58021, -1.58975, -1.56941, -1.51671, -1.45764, -1.38368,
  -1.29194, -1.19098, -1.08795, -0.998538, -0.901809, -0.773562, -0.672014,
  -0.625737, -0.595262, -0.557055, -0.501322, -0.432812, -0.38906, -0.341479,
  -0.226488, -0.0813971, 0.00971573, -0.0240986, -0.234482, -0.550708}
```

Just to confirm, the separated reluctance torque + PM torque should add up to be the same as the input torque

```
In[107]:= ListPlot[{X[tto], X[tfpm + tfpr], X[tin]}, Joined → True]
```



```
In[108]:= ListPlot[{X[tto], X[tfpm], X[tfpr], X[tcog]}, Joined → True, Frame → True,  
ImageSize → 500, FrameLabel → {"Mechanical Rotor Angle, Degrees", "Torque, Nm"},  
PlotRange → {-3, 5}, BaseStyle → {FontFamily → "Arial", FontSize → 14}]
```

